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STEPHEN JAUREGUI, JR.

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ON THE BROWN METHOD

OF SOLVING GAMES

by

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Lieutenant, United States Navy

Submitted in partial fulfillment of
the requirements for the degree of

MASTER OF SCIENCE

United States Naval Postgraduate School
Monterey, California

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ABSTRACT

The Brown method of solving zero sum two person games by a method of successive approximations was programmed for the NCR-102A Digital Computer. Game matricies up to order 8 x 8 were investigated, although the program could easily be extended to order 16 x 16 without leaving the magnetic drum, or to arbitrarily higher order games by also using magnetic tape. The problem of obtaining all strategies of a convex set of optimal strategies was solved in a number of cases and the concept of a complementary game was developed.

The Brown method was found to converge too slowly in most cases so that a modification of the method was used. In the Brown method, successive approximate strategies are developed for each player until a stage is reached in which the opposing strategies give equal values to the game (or give values sufficiently close), at which time (approximate) optimal strategies have been obtained. Julia Robinson has proved the convergence of the Brown method. The modification consists in comparing a maximum value of the game for Player I with a minimum value of the game for Player II at different stages of the iteration until these values are equal or sufficiently close to each other. The convergence of the new method follows from the convergence of the Brown method and the new method was found to be generally much more rapid.

The writer wishes to express his appreciation for the assistance and encouragement given him by Professor F.M. Pulliam of the U.S. Naval Postgraduate School in the investigation.

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INTRODUCTION

Two person zero sum games can be solved by a variety of methods, the common methods being graphical, matrix methods, linear programming, and the Brown approximation method. The last two methods are especially adaptable to computer techniques. The Brown method was investigated for two factors: speed of obtaining a solution and the ability to extract more than one solution, in case multiple solutions existed. The Brown method was found to be quite slow and to only give one solution for a single computer run; however, a variation on the method which is both faster and able to produce more than one solution on a single computer run was devised. This improved method should be a valuable tool in the solution of large games.

The sequence of topics to be discussed is as follows: first, a quick look at the nomenclature and the general idea of solving games; after this, the Brown method is discussed from the heuristic viewpoint. Next is an outline of the proof of the convergence of the Brown method, which is due to Julia Robinson. A brief discussion of the results obtained by the pure Brown method are then discussed for a few sample games. The next chapter discusses the improved program mentioned earlier, and this is followed by a list of techniques tried to extract more solutions when they existed and comments on the varying degrees of success of these methods. The following chapter is an analysis of a random 8 x 8 game, this, in turn, being followed in the last chapter by the actual program as used on the NCR-102A computer.

The appendix contains tables showing some of the results obtained on specific games which were solved in the course of studying the method.

CHAPTER I

BACKGROUND AND NOMENCLATURE

The theory of games was developed to handle problems involving conflicts of interest, such as in poker and warfare. In a military situation, a commander is required to weigh each of his own possible courses of action against each possible course of action of the enemy to properly decide which course of action he should take. The commander might, by evaluating the possible outcomes as they effect him, assign numbers to these outcomes and formulate a table (see Fig. 1) which tabulates these outcomes in matrix form. The larger the entry, the more favorable is the outcome to the friendly troops.

Enemy Strategies

Friendly Strategies		$\boxed{1}$	$\boxed{2}$	$\boxed{3}$	$\boxed{4}$
	1	-3	2	4	-2
	2	3	-1	1	2
	3	1	-2	-3	1

Figure 1

The commander desires to choose his course of action so as to maximize his outcome, while the enemy wishes to choose his strategy so as to minimize this outcome to his opponent. The clash could be a one-shot affair or a situation which might repeatedly arise, such as a fighter attack on a bomber.

When two opponents are involved in a conflict in which each has a definite number of possible strategies and the outcome for each pair of opposing strategies is known, it

[illegible]

[illegible]

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1

is said to be a two person game. The game is said to be zero sum if the sum of the outcomes for the opponents is zero; that is, each player's loss is the other player's gain. A matrix is used in which the rows correspond to strategies for what is called Player I, and the columns correspond to strategies for what is called Player II; the element a_{ij} in the i^{th} row and j^{th} column is the payoff to Player I if he uses his i^{th} strategy and Player II uses his j^{th}.

Player II Strategies

$$\begin{array}{ll}
 & \begin{array}{cccccc} \boxed{1} & \boxed{2} & \boxed{3} & \boxed{4} & \cdots\cdots & \boxed{n} \end{array} \\
\text{Player I Strategies} \quad \begin{array}{c} 1 \\ 2 \\ \cdot \\ \cdot \\ m \end{array} & \left\| \begin{array}{cccccc} a_{11} & a_{12} & a_{13} & a_{14} & \cdots\cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & a_{24} & \cdots\cdots & a_{2n} \\ \cdot & \cdot & \cdot & \cdot & \cdots\cdots & \\ \cdot & \cdot & \cdot & a_{rs} & \cdots\cdots & \\ a_{m1} & a_{m2} & a_{m3} & a_{m4} & \cdots\cdots & a_{mn} \end{array} \right\| = A
\end{array}$$

Figure 2

If the element a_{rs} is the minimum element of the r^{th} row and the maximum of the s^{th} column, then Player I is guaranteed the amount a_{rs} if he picks his "pure strategy" r and Player I can receive no more than a_{rs} if Player II picks his "pure strategy" $\boxed{s}$. In this particular case, Player I's "optimal strategy" is r , Player II's "optimal strategy" is $\boxed{s}$, and the "value of the game" is a_{rs}. a_{rs} is called a saddle point, which is a desirable feature in a one-shot game. All games of "perfect information," such as checkers, have saddle points.

If there is no saddle point in the game matrix, then

[illegible]

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[illegible]

if Player I uses his strategy (i) with probability x_i and Player II uses his strategy $\boxed{j}$ with probability y_j, they are using "mixed strategies":

$$0 \leq x_i \leq 1 \qquad \sum x_i = 1$$

$$0 \leq y_j \leq 1 \qquad \sum y_j = 1$$

If $X = \begin{Vmatrix} x_1 \\ x_2 \\ x_3 \\ \cdot \\ \cdot \\ x_m \end{Vmatrix}$ and $Y = \begin{Vmatrix} y_1 \\ y_2 \\ y_3 \\ \cdot \\ \cdot \\ y_n \end{Vmatrix}$ then

$X'AY$ is the expectation of Player I if Player I uses mixed strategy X and Player II uses mixed strategy Y. Player I wants to maximize the expectation while Player II wishes to minimize it. Player I will receive at least $\min\limits_{Y} X'AY$ so Player I selects X so as to get $\max\limits_{X} \min\limits_{Y} X'AY$. On the other hand, Player II can hold Player I to at most $\max\limits_{X} X'AY$ so he picks Y to realize $\min\limits_{Y} \max\limits_{X} X'AY$.

John Von Neumann's fundamental theorem of game theory states that $\max\limits_{X} \min\limits_{Y} X'AY = \min\limits_{X} \max\limits_{Y} X'AY = v$ (value of the game). Hence, there exists a mixed strategy X* which is such that the expectation of Player I will be at least v, and there exists a mixed strategy Y* such that the expectation of Player I will be at most v. If Player I uses X*

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and Player II uses Y*, the expectation of Player I will be exactly v, the "value of the game." X* and Y* are called "optimal mixed strategies." The "solution" of the game A consists of determining possible X*, Y*, and v. The following facts are stated for later use in this paper:

1. If a constant c is added to each element a_{ij} of the game matrix A, the value of the game is increased by c, but the optimal strategies are not changed. Therefore, without loss of generality game matricies consisting only of non-negative elements need be considered in this paper.

2. If $x_i \neq 0$ in X* then i yields Player I the value of the game if Player II uses Y*; similarly, if $y_j \neq 0$ in Y*, the $\boxed{j}$ holds Player I to the value of the game if used against an X*.

CHAPTER II

THE BROWN METHOD OF FICTITIOUS PLAY

The Brown Method of Fictitious Play is a method of successive approximations to the solution of a game.

The method is as follows:

a. Player I picks an arbitrary pure strategy for his first play.

b. Player II picks a pure strategy to minimize what Player I would get from his previous play.

c. Player I then picks a pure strategy to maximize what he would get against the pure strategy Player II has just used and adds this pure strategy to what he picked before.

d. Player II next picks his best strategy against the sum of Player I's past strategies. This process is then repeated from the fictitious play of one player back to the fictitious play of the other, over and over again.

e. Player I at play N forms the sum of his first N pure strategies and divides the minimum number of this "row sum" by N to get the "value of the game from below at play N" symbolized by $\underline{v}(N)$, or more simply, $\underline{v}$. This is the approximate value of the game to himself through play N. The number of times each pure strategy i has been selected is divided by N to give x_i^* for the approximate optimal strategy $X^*(N)$.

f. Player II computes at his Nth play the sum of his first N pure strategies and divides the maximum number in the "column sum" by N to get an approximate "value of the

game from above at play N", symbolized by $\bar{v}(N)$, or more simply $\bar{v}$, which is the amount that Player II must pay to Player I, if Player II uses his approximate optimal strategy $Y^*(N)$ which is formed by dividing the number of times each pure strategy has been picked, by N. This forms y_j^* whose sum, of course, is equal to one.

g. At the end of each pair of plays $\bar{v}(N) - \underline{v}(N)$ is computed and when this value becomes less than a predetermined amount, the procedure is terminated. (See Chapter III for the convergence of the procedure.)

Consider the simple 3 x 3 game:

$$
\begin{array}{ccc}
 & & \text{Player II} \\
 & & \begin{array}{ccc} \boxed{1} & \boxed{2} & \boxed{3} \end{array} \\
\text{Player I} & \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \left\| \begin{array}{ccc} 2 & 5 & 5 \\ 6 & 3 & 4 \\ 0 & 6 & 3 \end{array} \right\|
\end{array}
$$

Suppose that Player I picks 1 as his first pure strategy:

$$
\begin{array}{cccc}
 & \boxed{1} & \boxed{2} & \boxed{3} \\
1 & 2 & 5 & 5
\end{array}
\qquad \underline{v}(1) = 2
$$

Player II will now take $\boxed{1}$ since 2 is the minimum of the three choices.

$$
\begin{array}{cccc}
 & 1 & 2 & 3 \\
\boxed{1} & 2 & 6 & 0
\end{array}
\qquad \bar{v}(1) = 6
$$

Player I will now choose 2 since 6 is the maximum element in 1 :

$$
\begin{array}{cccc}
 & \boxed{1} & \boxed{2} & \boxed{3} \\
1 \text{ plus } 2 & 8 & 8 & 9
\end{array}
\qquad \underline{v}(2) = \frac{8}{2} = 4
$$

Player II now has a choice between (1) and (2) since they both equal 8; as a rule, for convenience, the first of equals will always be picked.

The following table shows the outcome for 12 plays, while in Appendix A-12 this same game is carried out for 30 iterations.

Play N	Strat. i(N)	Row Sum 1	2	3	$\frac{\text{Min } R_i}{N}$ $\underline{v}$	j(N)	Col. Sum 1	2	3	$\frac{\text{Max } C_j}{N}$ $\bar{v}$	$\bar{v} - \underline{v}$
1	1	2	5	5	2.	1	2	6	0	6.	4.
2	2	8*	8	9	4.	1	4	12	0	6.	2.
3	2	14	11	13	3.67	2	9	15	6	5.	1.33
4	2	20	14	17	3.5	2	14	18	12	4.5	1.
5	2	26	17	21	3.4	2	19	21	18	4.2	.8
6	2	32	20	25	3.33	2	24*	24	24	4.	.67
7	1	34	25	30	3.57	2	29	27	30	4.28	.71
8	3	34	31	33	3.88	2	34	30	36	4.5	.62
9	3	34	37	36	3.64	1	36*	36	36	4.	.36
10	1	36	42	41	3.6	1	38	42	36	4.2	.6
11	2	42	45	45	3.8	1	40	48	36	4.37	.57
12	2	48	48	49	4.	1	42	54	36	4.5	.5

*The rule of selection is if two rows or columns are equally desirable, the program selects the first.

Table 1

An examination of play 9 shows that through play 9, Player I is using a mixed strategy of (2/9, 5/9, 2/9); the value of the game to Player I is at least 3.64. Player II at play 9 is using a mixed strategy of (3/9, 6/9, 0) and the value of the game to him (the most he can lose) is 4.

CHAPTER III

ON THE ROBINSON PROOF

G. W. Brown conjectured in 1949 (Ref. a) that the method of solving rectangular games which bears his name would converge and Julia Robinson proved the convergence in an article published in 1951 (Ref. b).

Mrs. Robinson assumes the game matrix to be an $m \times n$ matrix $A = (a_{ij})$. She uses $A_{i.}$ and $A_{.j}$ to represent, respectively, the ith row and the jth column of A, lets the max of a vector be a maximum element of it, and the min of a vector be a minimal element of it. She defines a vector system for A as follows:

The system (U, V) in which U is a sequence of n dimensional vectors U(0), U(1), U(2).....U(N).....and V is a sequence of m dimensional vectors V(0), V(1).....V(N)... is called a vector system for A if:

$$\min U(0) \qquad \max V(0) \qquad \text{and}$$

$$U(N+1) = U(N) + A_{i.}$$

$$V(N+1) = V(N) + A_{.j}$$

where i and j are such that $v_i(N) = \max V(N)$ and either $u_j(N) = \min U(N)$ or $u_j(N+1) = \min U(N+1)$

If j is defined by $u_j(N) = \min U(N)$, then for each N, U(N + 1) and V(N + 1) are obtained simultaneously from U(N) and V(N). If j is defined by $u_j(N+1) = \min U(N+1)$, then the U's and V's are obtained alternately. The alternate method is used in this thesis since it converges more rapidly than the simultaneous method. U(0) and V(0) may be defined arbitrarily and if they are both null. U(N)/N and

$V(N)/N$ are, respectively, weighted averages of the rows and columns of A which are used in $U(N)$ and $V(N)$.

$U(N)$ is the sum of rows of A which Player I computes at stage N, and $\min U(N)/N$ his approximate value. $V(N)$ and $\max V(N)/N$ have similar significance for Player II.

Julia Robinson proved the basic theorem:

If (U, V) is a vector system for A, then

$$\lim_{N \to \infty} \min U(N)/N = \lim_{N \to \infty} \max V(N)/N = v$$ the value of the game A.

From the basic definitions

$$\min U(N)/N \leq v \leq \max V(N')/N'$$

If for some N and N' the equalities hold, then optimal strategies are known as well as the value of the game v. Hence, it may not be necessary to take the limit in order to get a solution to the game.

Mrs. Robinson proves the theorem through four lemmas:

Lemma 1. If (U, V) is a vector system for A, then

$$\liminf_{N \to \infty} \frac{\max V(N) - \min U(V)}{N} \geq 0$$

Lemma 2. Given a vector system (U, V) for A, then if all the rows and columns of A are eligible (see note) in the interval $(S, S + N)$

$$\max U(S + N) - \min U(S + N) \leq 2aN$$

$$\text{and} \quad \max V(S + N) - \min V(S + N) \leq 2aN$$

where $a = \max a_{ij}$

Note: "The ith row is eligible in $(S, S + N)$ if for an N' such that

$$S \leq N' \leq S + N \qquad v(N') = \max V(N').$$

similarly for the jth column with min $U(N'')$."

Lemma 3. If all the rows and columns of A are eligible in $(S, S + N)$ for a given vector system (U, V) then $\max V(S + N) - \min U(S + N) \geq 4aN$.

From Lemma 2 and Lemma 3, we have Lemma 4:

Lemma 4. To every matrix A and $\mathcal{E} > 0$, there exists N_0 such that for any vector system (U, V)

$$\max V(N) - \min U(N) < \mathcal{E} N \qquad N \geq N_0$$

From Lemma 's 1 and 4 it follows that

$$\lim_{N \to \infty} \frac{\max V(N) - \min U(N)}{N} = 0$$

From basic game theory

$$\limsup_{N \to \infty} \frac{\min U(N)}{N} \leq v \quad \text{and} \quad \liminf_{N \to \infty} \frac{\max V(N)}{N} \geq v$$

The theorem follows.

CHAPTER IV

SOME RESULTS OF THE BROWN METHOD

Several different games will be examined in some detail using the Brown method. Curves of successive values from both above ($\overline{v}$) and below ($\underline{v}$) will be plotted and, for one game, a brief plot of the current mixed strategy versus the number of play.

Three different games illustrate three different possibilities in the way that a game converges to its value. The first game to be examined is the one in the example in Chapter II.

$$\begin{Vmatrix} 2 & 5 & 5 \\ 6 & 3 & 4 \\ 0 & 6 & 3 \end{Vmatrix}$$

From the curve (Fig. 3), it is seen that although the values $\overline{v}$ and $\underline{v}$ are at times equal to the value of the game which is four, but not at the same play, convergence is an extremely slow process taking approximately 2500 iterations to converge to 4 decimal-place accuracy. However, looking at Fig. 3a, which is purely a permutation of the game matrix and therefore essentially the same game, convergence takes place in exactly twelve plays to the exact value of the game. All 36 possible permutations of rows and columns are carried out either to an exact solution or for 30 iterations in Appendix A; and examination of these permutations shows that six have exact values and that the other 30 all converge slowly and cannot, in a finite number of plays, have exact convergence by the Brown method since unwanted strategies

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[illegible]

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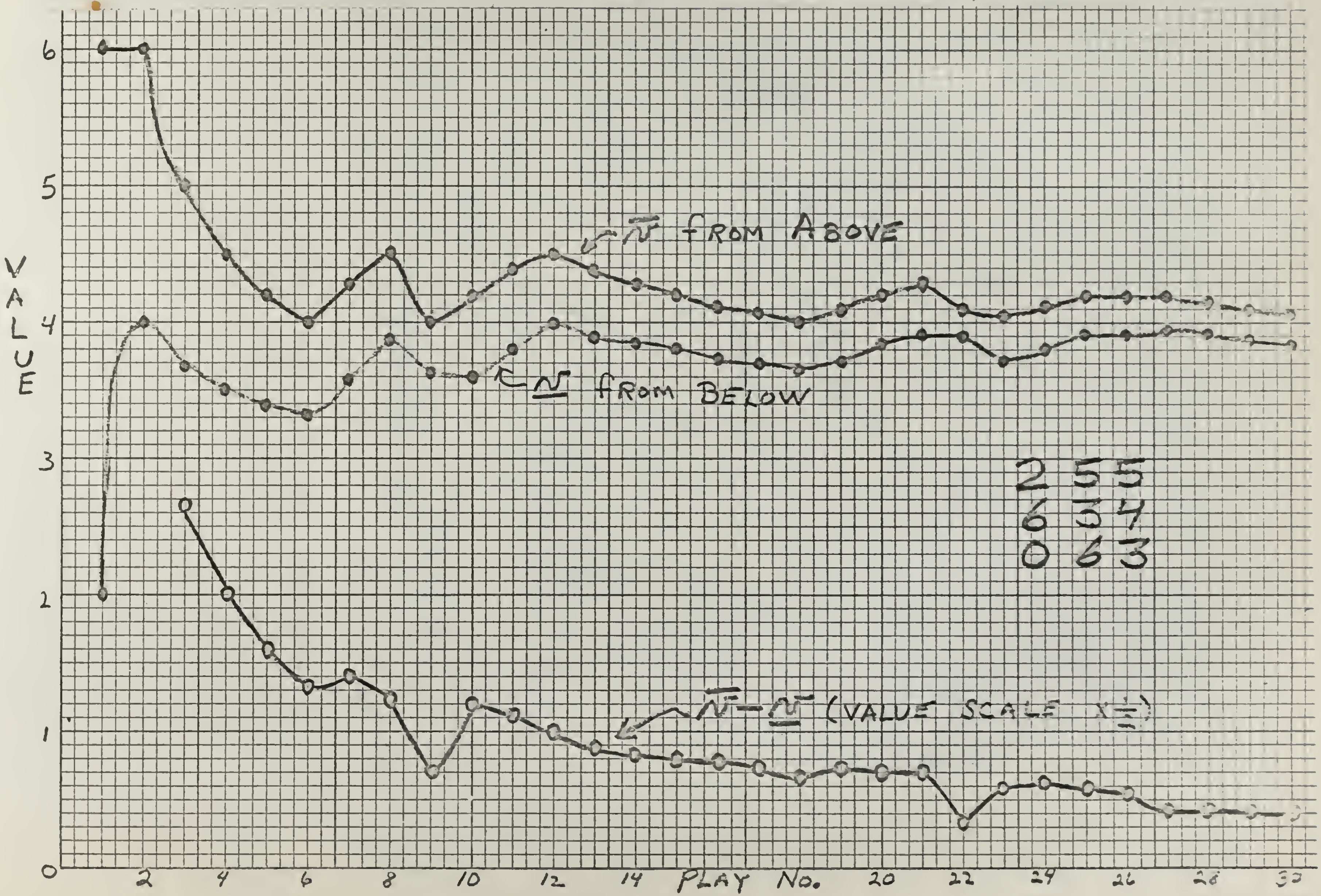
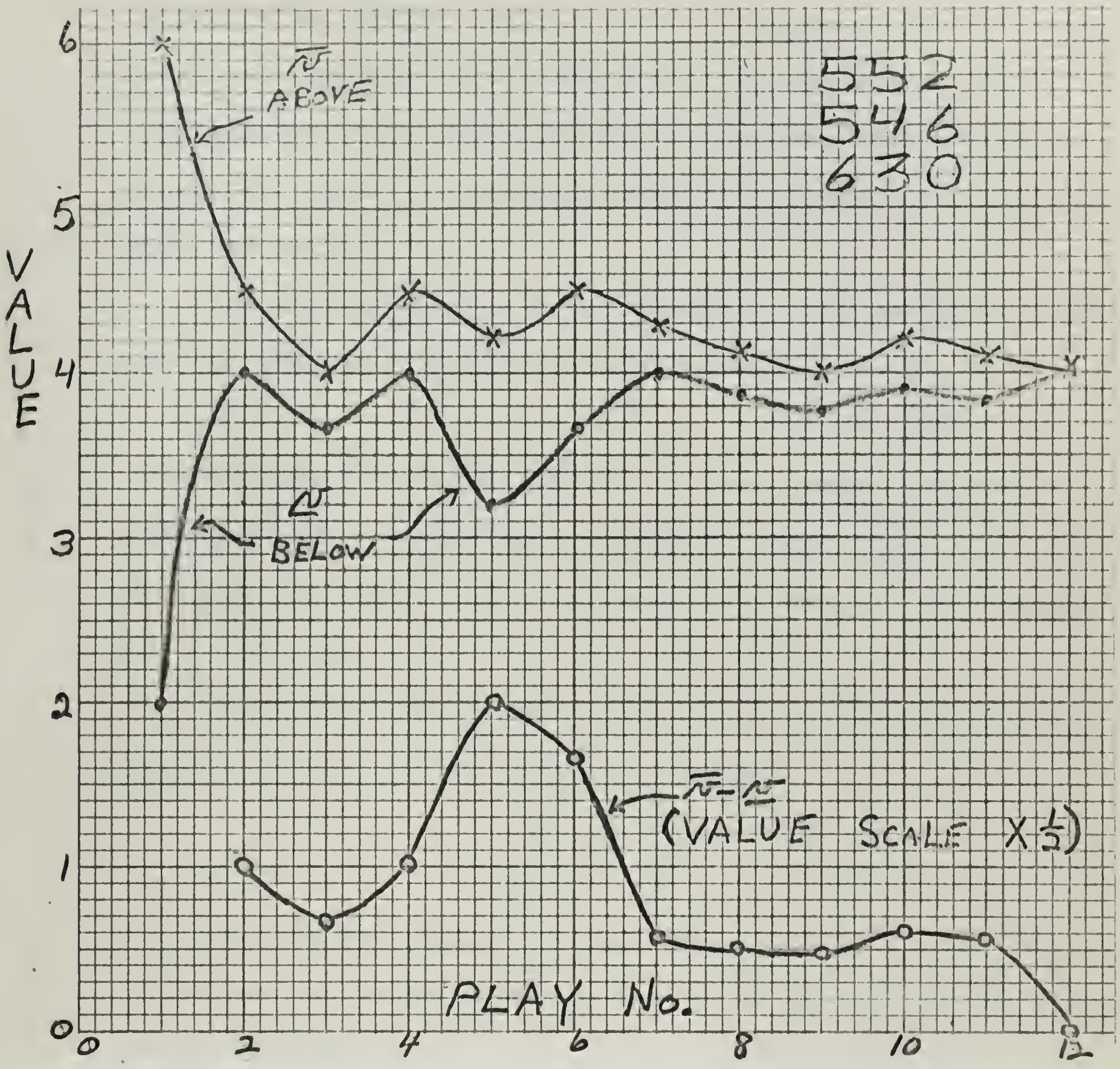


A Case I Game

Figure 3



A Case I Game With Brown Solution

Figure 3a

have been selected for Player II. It would be very desirable to find what determined the rapid convergence for the 6 permutations and the slow convergence for the rest. This investigator was unable to see any pattern that might be generally useful for picking the right permutation to solve the game best.

The second game to be discussed is the permuted complementary game of the first illustration $\begin{Vmatrix} 5 & 4 & 3 \\ 5 & 3 & 6 \\ 2 & 6 & 0 \end{Vmatrix}$

In this game $\bar{v}$ is occasionally v, but $\underline{v}$ never becomes v although max $\underline{v}$ approaches v asymptotically (Fig. 4). It required 536 plays for this game to converge to 3 decimal-place accuracy.

The third case is when both min $\bar{v}$ and max $\underline{v}$ converge asymptotically to the value of the game.

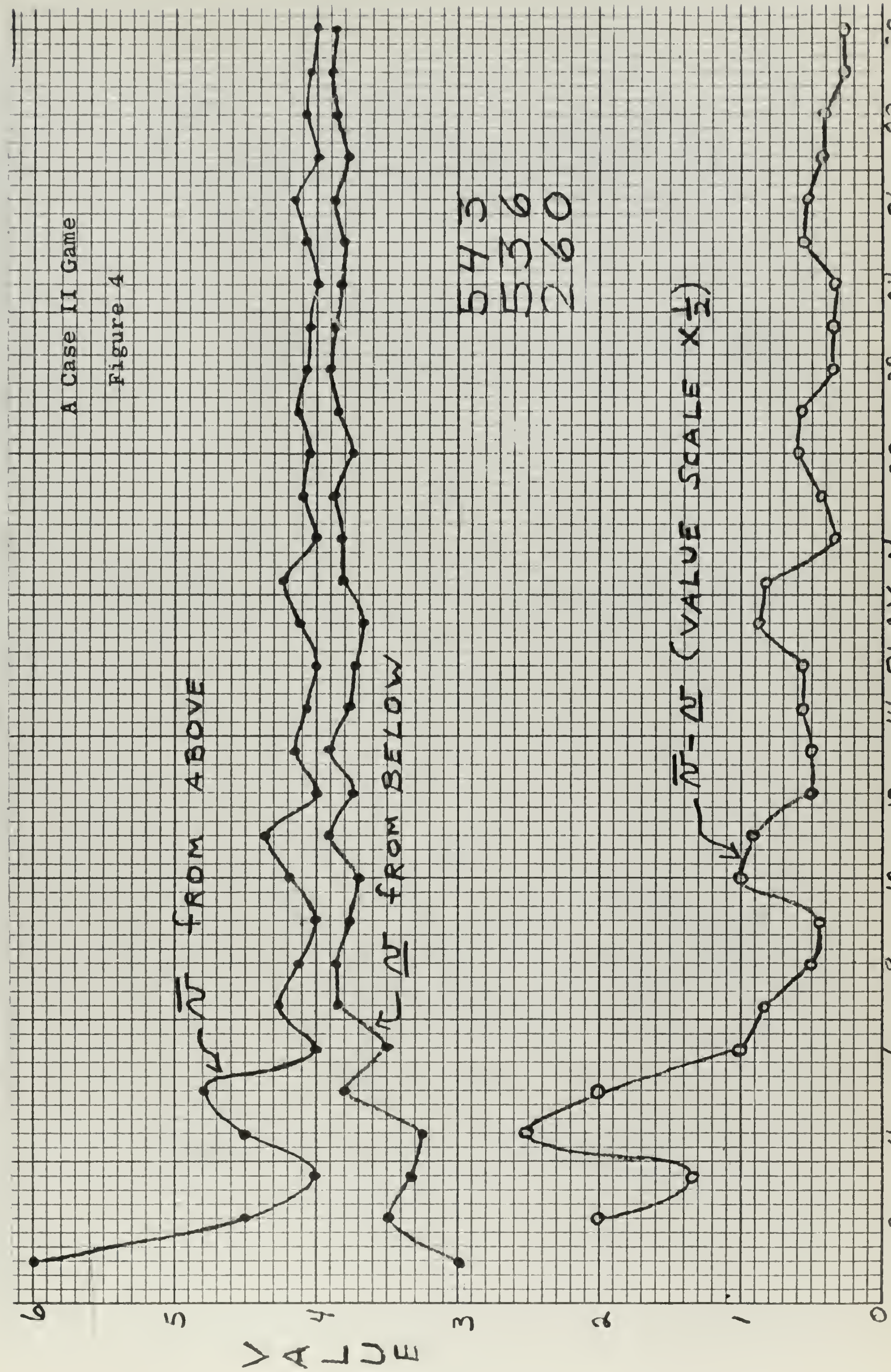
$$\begin{Vmatrix} 6 & 7 & 1 \\ 6 & 3 & 2 \\ 2 & 4 & 5 \end{Vmatrix}$$

the graph of the values on this game is Fig. 7, and the table of values and strategies for the first 30 iterations is given in the appendix.

Another example of a Case I game is illustrated in Fig. 8; this is a game in which the optimal strategy of each player is (.2, .2, .2, .2, .2). The mixed strategies versus the number of play is given for a portion of the convergence cycle. It is noted that a type of "inertia" seems to exist in the selection of pure strategies to add to the mixture at each play. For those that are below their share, they keep

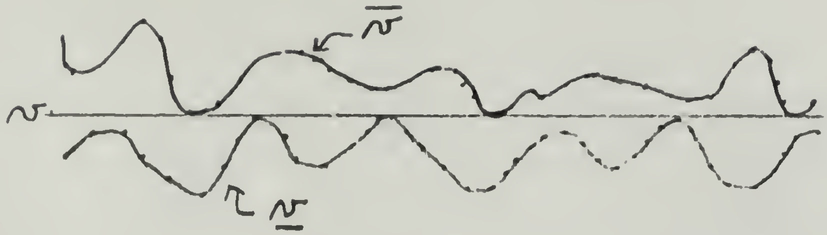
being added to until they exceed the value and then are allowed to fall off again.

The terms Case I, Case II, and Case III will be used throughout this paper to type the three types of games discussed in this section. Fig. 5 re-emphasizes these definitions.



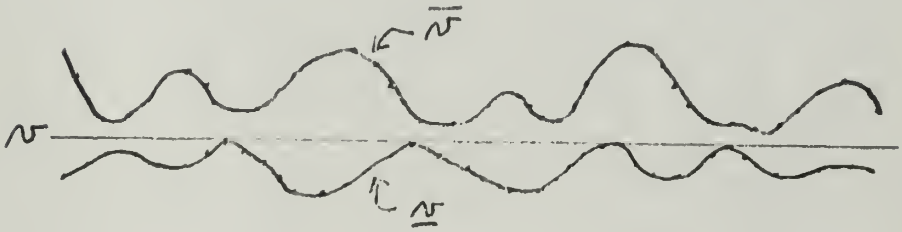
A Case II Game

Figure 4

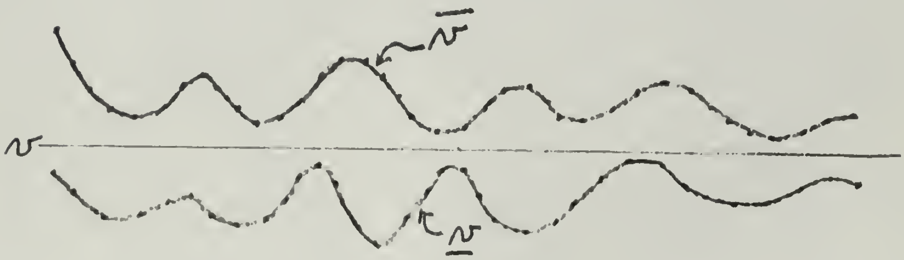


Case I Game

Number of play is plotted horizontally, value is plotted vertically.



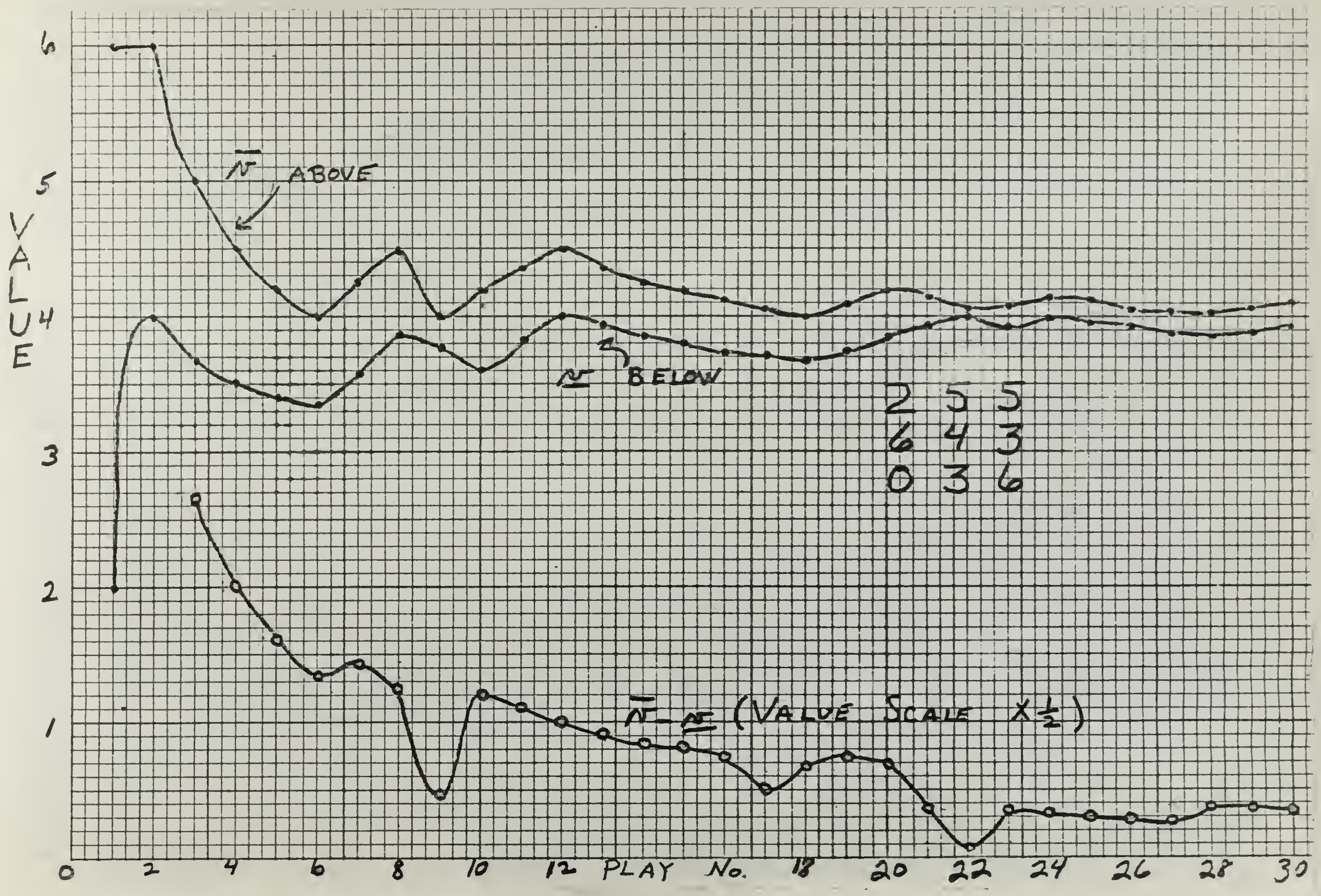
Case II Game



Case III Game

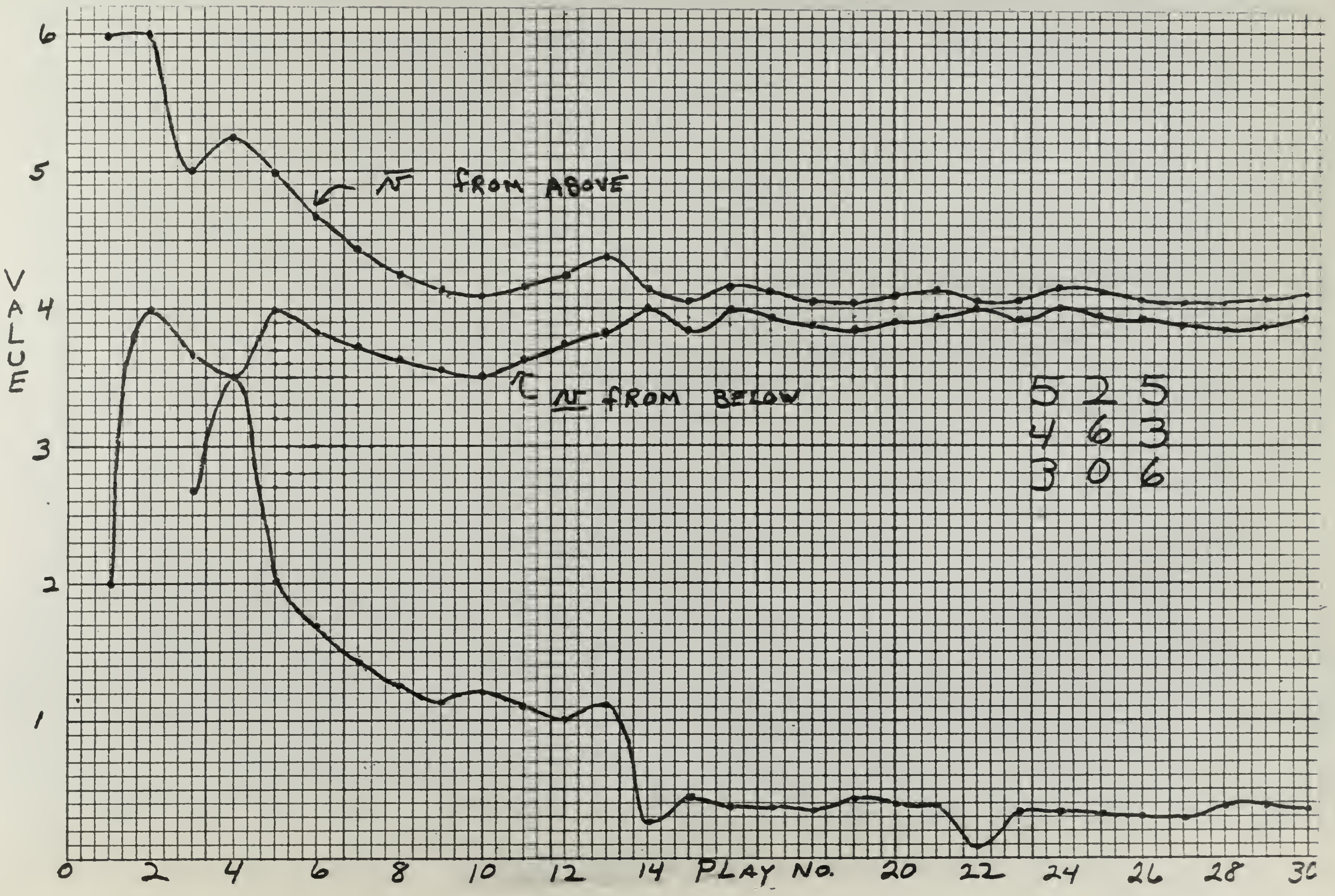
A Summary of the Three Cases of Games

Figure 5



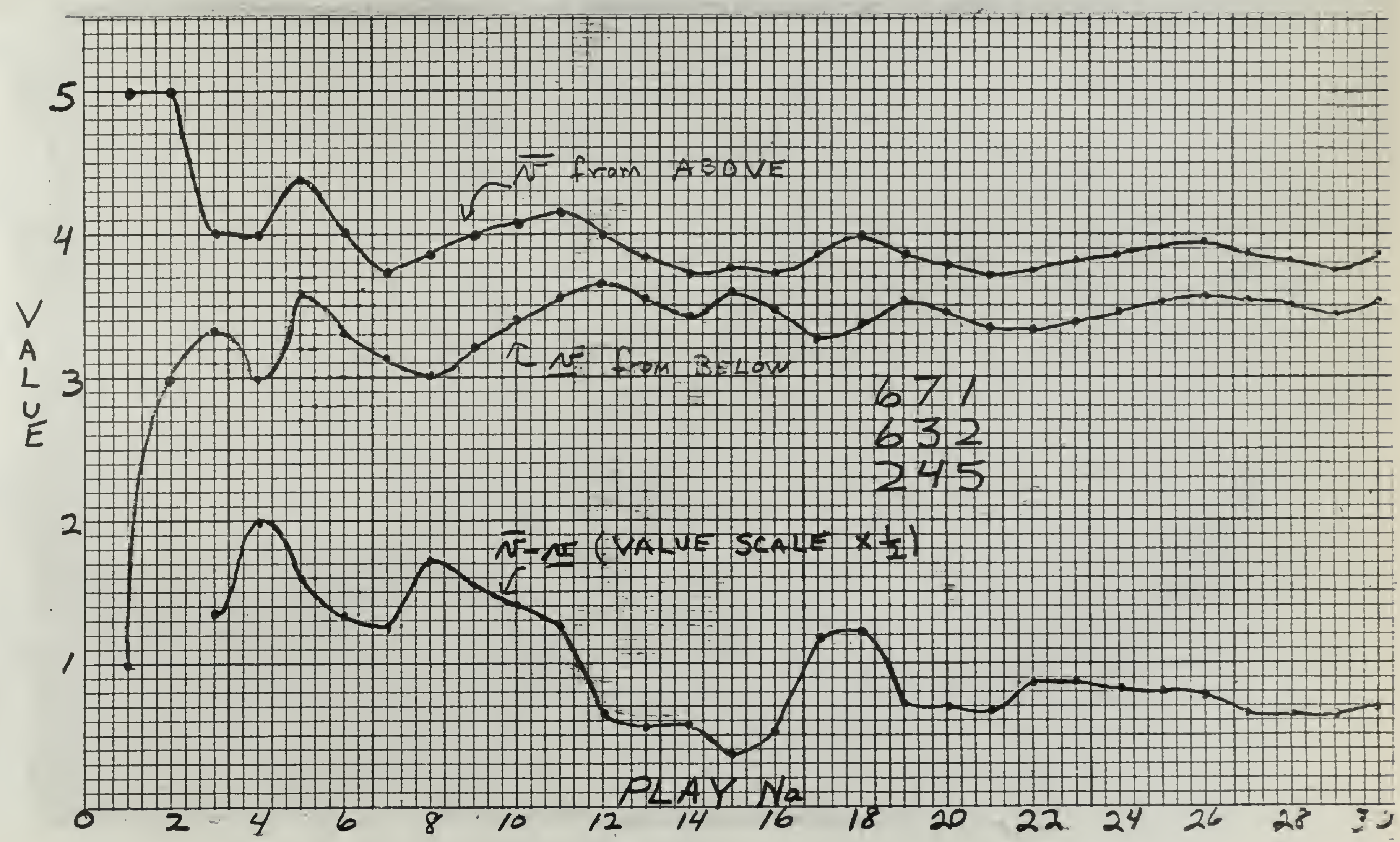
Another Case I Game

Figure 6



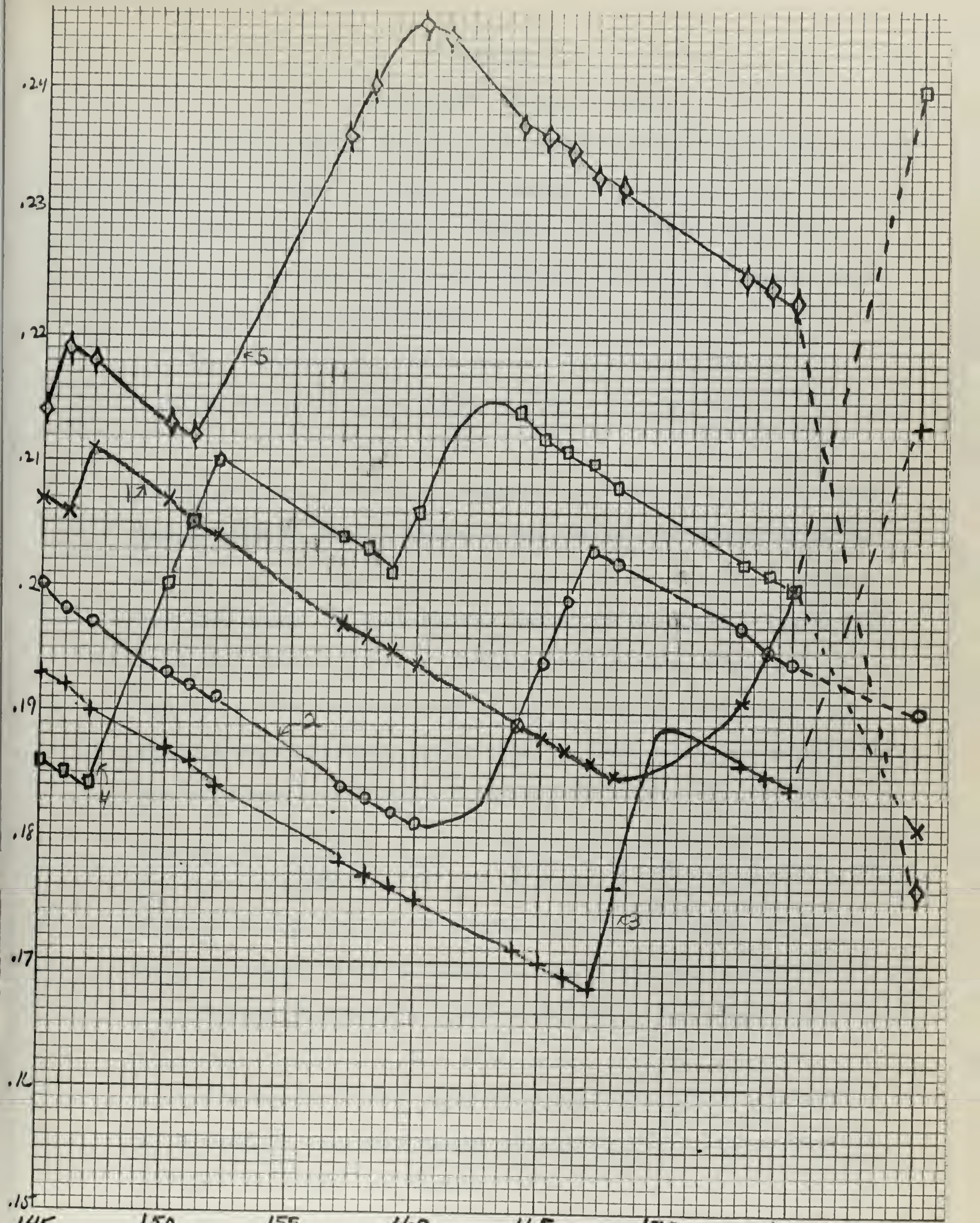
Another Case II Game

Figure 6a



A Case III Game

Figure 7



The optimal strategy is (.2, .2, .2, .2, .2)

Strategy at Play N vs Play N

Figure 8

CHAPTER V

IMPROVEMENT ON THE METHOD

As was seen in Chapter IV, although the Brown method did converge as expected, in many games it was very slow and yielded only one solution. Hence, a modification of the method was sought which might speed up the convergence and also produce more solutions.

In reference to the game

$$\begin{Vmatrix} 2 & 5 & 5 \\ 6 & 3 & 4 \\ 0 & 6 & 3 \end{Vmatrix}$$

a glance at the chart in Chapter II shows a very interesting result in that in play 2 the value for Player I is 4, and in step 6 the value for Player II is 4. Since these two are equal, 4 must be the value of the game. Again, in step 12, Player I has a value of 4--as does Player II in step 18. The next procedure is to verify that these are truly solutions, so a test will be made of these strategies. The strategy for Player I at play 2 is $X^* = (\frac{1}{2}, \frac{1}{2}, 0)$

$$(X'A)' = \begin{Vmatrix} 4 \\ 4 \\ 4\frac{1}{2} \end{Vmatrix} \qquad \text{or} \qquad \begin{matrix} \frac{1}{2} \times 2 + \frac{1}{2} \times 6 + 0 \times 0 & = \\ \frac{1}{2} \times 5 + \frac{1}{2} \times 3 + 0 \times 6 & = \\ \frac{1}{2} \times 5 + \frac{1}{2} \times 4 + 0 \times 3 & = \end{matrix} \begin{Vmatrix} 4 \\ 4 \\ 4\frac{1}{2} \end{Vmatrix}$$

therefore, $(\frac{1}{2}, \frac{1}{2}, 0)$ is a solution for Player I; and, as will be shown later in Chapter VII, this is a basic solution.

Trying Player I's other solution at play 12, the optimal strategy is (3/12, 7/12, 2/12); this gives values of 4, 4, $4\frac{1}{2}$ respectively against Player II's pure strategies 1, 2, and 3. So this again satisfies the criteria for an optimal strategy for Player I.

Player II has the strategy of (1/3, 2/3, 0) at play 6, where the value from above is 4. A check of this in the summation $\sum a_{ij}y_j \leq v$ shows this satisfies the criteria for an optimal strategy for the minimizing player.

Play 18 gives the strategy (6/18, 12/18, 0) which is again (1/3, 2/3, 0) and, therefore, yields no new solution for Player II. In fact, this solution is unique for him.

The Brown method was then modified to pick off the maximum value of $\underline{v}$ (from below) for Player I and the minimum value of $\overline{v}$ (from above) and retain the mixed strategies for these values until a new maximum or minimum value was generated. Each new value generated and the corresponding strategies were made available for print out. Whenever $\overline{v} = \underline{v}$, a certainty of optimality exists. This modified method not only gives a much faster solution but, as a bonus, yields more than one solution as was seen in the example above.

In case max $\underline{v}$ (or min $\overline{v}$) repeats itself in all 12 octal digits on the computer, one can be almost sure that it is the value of the game as the probability of two twelve-digit numbers being identical except in the actual limit is very small (a check of several thousand iterations on various games found no repeats, except in the first two or three iterations).

In the Case II example given in Chapter IV, convergence to 3 decimal-place accuracy came in 257 iterations, as compared to 536 plays of the Brown method. This is a saving in

time of over 50%.

Two graphs, Fig. 9 and Fig. 10, compare $\overline{v}-\underline{v}$ of the Brown method with that of the improved method. The first 25 plays are shown in Fig. 9, while the value differences from iteration 265-290 are shown in Fig. 10. It is to be noted that although the $\overline{v}-\underline{v}$ for the first few iterations were quite irregular, the process seems to have settled down to a cyclic process after many iterations as seen in the second graph. Also, the values for the improved method are considerably under those in the regular method and will, therefore, give much faster convergence.

Of the 36 permutations of the Case I game illustrated in Chapter II, 12 games fall into Case I category and have exact solutions in a very few iterations as compared to only 6 games for the Brown method. All the rest of the 24 permutations are class II games and their exact solutions can normally be found by methods to be discussed later in just a few iterations; however, even letting them compute themselves out to the end result would still save about 50% on time.

A check of the six additional permutations which give exact solutions in the improved method (A-7 through A-12) shows that each of these games picks up an undesirable strategy and can never converge exactly by the Brown method in a finite number of plays (play 26 in A-12, for example).

A little ingenuity can reduce the time required even more. Consider again the Case II illustration; refer to A-37 which is a print out of the first 30 plays of the game.

It is seen that at play 6, Player II has a repeat value of 4 for the value of the game from above; and at play 7, Player I receives a new max value from below. Since the first play of Player I was arbitrary, subtract this row from the row sum of all his strategies

$$(29, 28, 27) - (5, 4, 3) \text{ equals } (24, 24, 24)$$

Dividing this by 6, which is now the number of plays since one play of Player I was deleted, a value of 4 is now obtained, giving a strategy of (0, 2/3, 1/3) which is optimal.

The number of iterations involved for solution is now 7 instead of the 256 of the improved method sans ingenuity or the 536 plays of the Brown method. It is to be noted, however, if an attempt were made to use Player I's max at play 8 or 11 the method would have failed, while at play 13 it would have succeeded again. Since the computer is operating all this time, there is no loss in time to try a few guided guesses.

Two Case III games were solved by the Brown method and its modification. Convergence by the new method required approximately 70% of the time required by the Brown method.

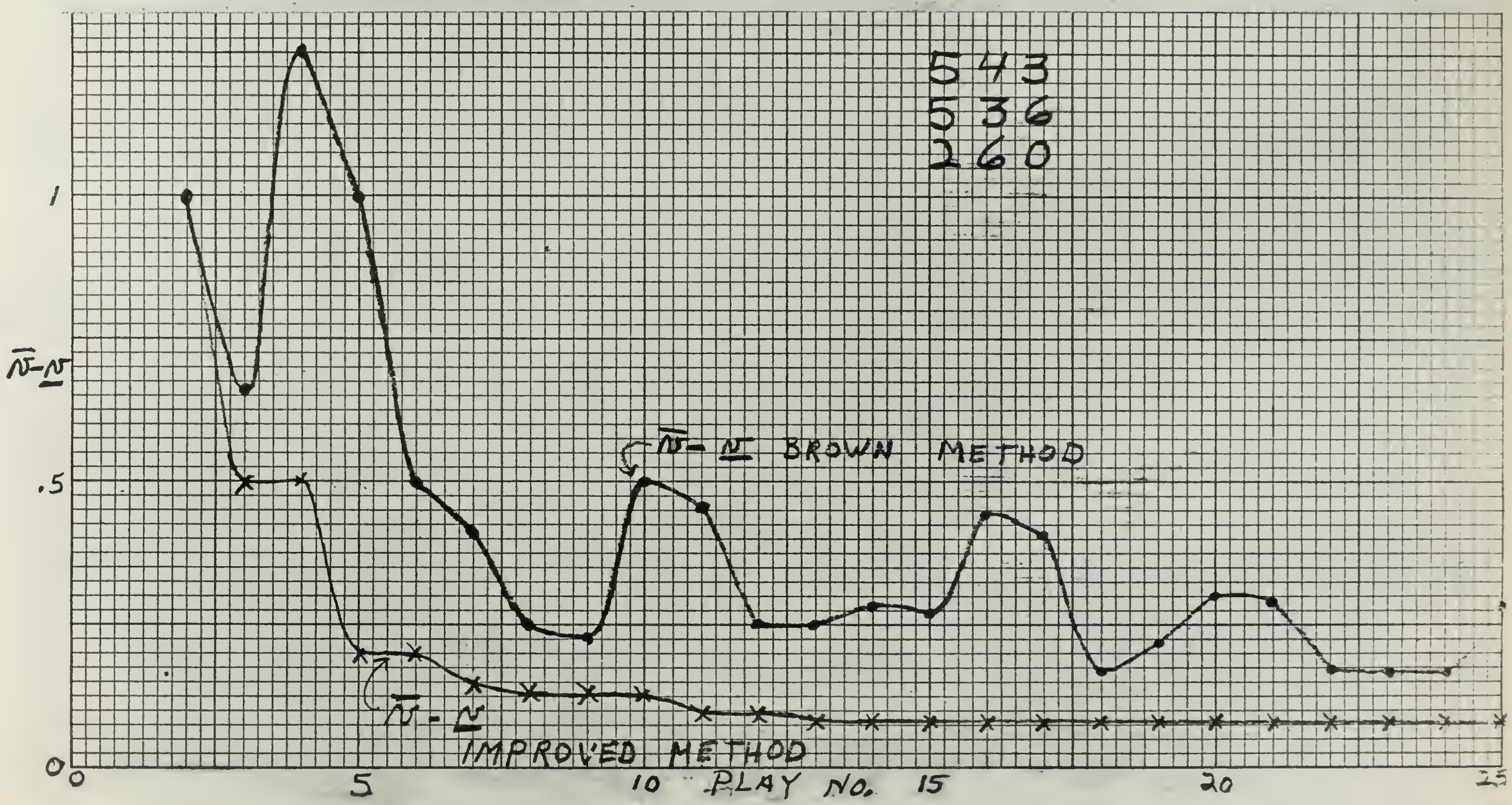
Another example of the superiority of the improved method is the game

$$\begin{Vmatrix} 3 & 1 & 4 & 1 & 6 \\ 6 & 3 & 1 & 4 & 1 \\ 1 & 6 & 3 & 1 & 4 \\ 4 & 1 & 6 & 3 & 1 \\ 1 & 4 & 1 & 6 & 3 \end{Vmatrix}$$

This game converged to an exact value by the improved method in only 40 iterations, while the Brown method used 1340 iterations to converge to only 2 place decimal accuracy and

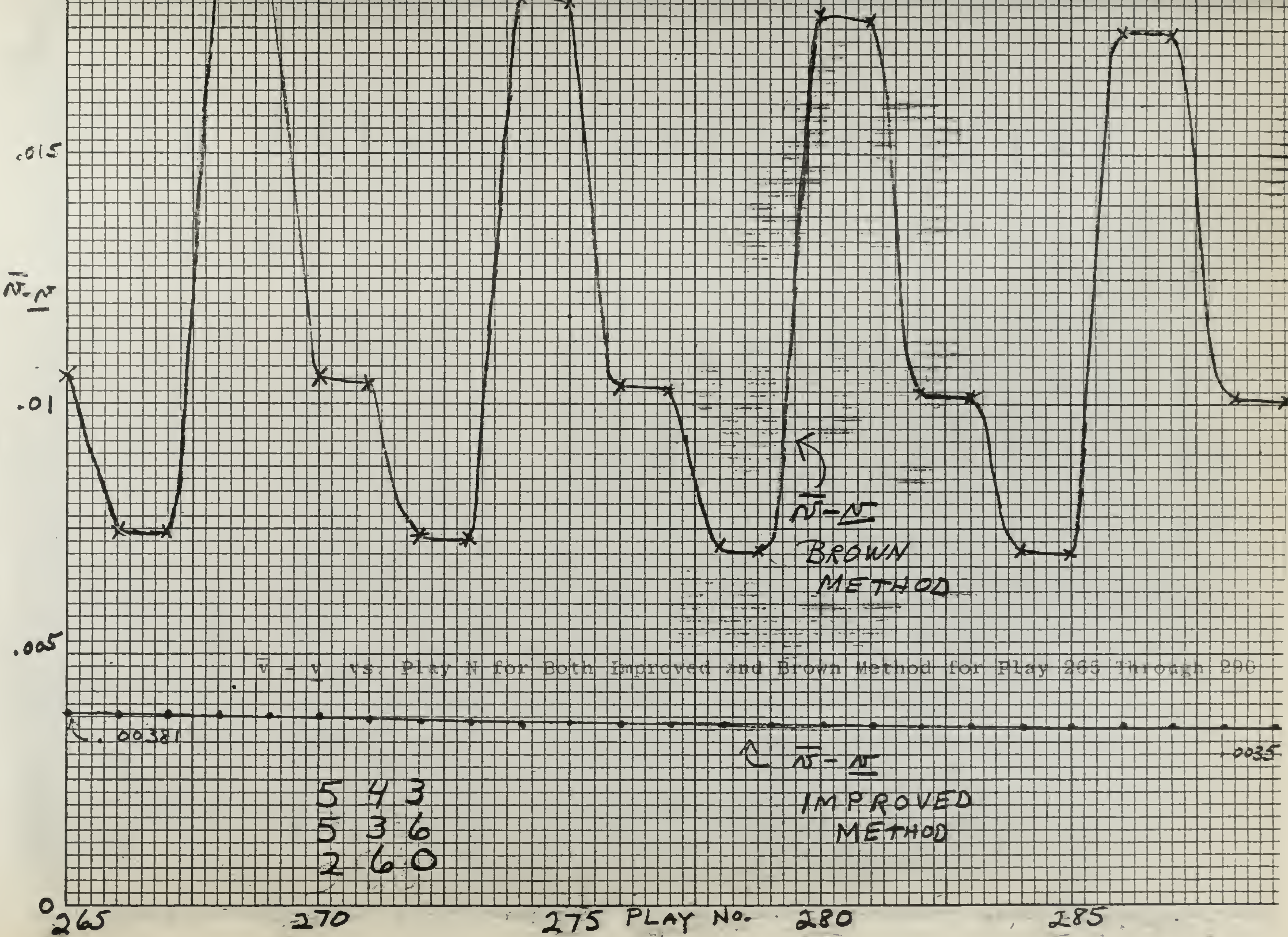
the value of the game is three. Since this is a symmetric game, the optimal strategies for both players are (.2, .2, .2, .2, .2). Fig. 8 shows the strategies actually being used at various plays from 145 to 175 by Player I; and with a sample again taken at iteration number 221, this quite clearly demonstrates the inertia in the Brown method, which is quite bad, even after as many iterations as these.

In summary, the improved method in Case I games is vastly superior to the older method. For Case II games, with a little hand manipulation and thought, the new method is almost as effective as in the Case I games. For Case III games, the improvement is marked but the process still seems quite slow.



$\bar{v}-\underline{v}$ vs. Play N for Both Improved and Brown Method for Play 1 Through 25

Figure 9



$\bar{v} - \underline{v}$ vs. Play N for Both Improved and Brown Method for Play 265 Through 290

Figure 10

CHAPTER VI

METHODS OF EXTRACTING OTHER OPTIMAL STRATEGIES

Various methods were tried to extract more than one optimal strategy in case it existed. The first method attempted was to permute the rows and columns of the game matrix. An examination of A-1 through A-6 shows the pure Brown method always led to the same exact solution; therefore, the permutations were not too successful with the pure Brown method.

The next attempt made was to add a small value to one row or to one column; this was accomplished by multiplying the matrix by ten and then adding one to the selected row or column. This seemed to have no effect on the solution in the games on which it was tried. The amount of the small addition to the row or column was increased until a change in optimal strategy was obtained. However, the fault now was that the new solution seemed to have no relation to the original game.

The inverse game was also used in an attempt to extract more optimal strategies. An inverse game is where the roles of Player I and Player II are reversed by transposing the game and then multiplying the elements by minus one. The results on the inverse game were identical to those on the original game.

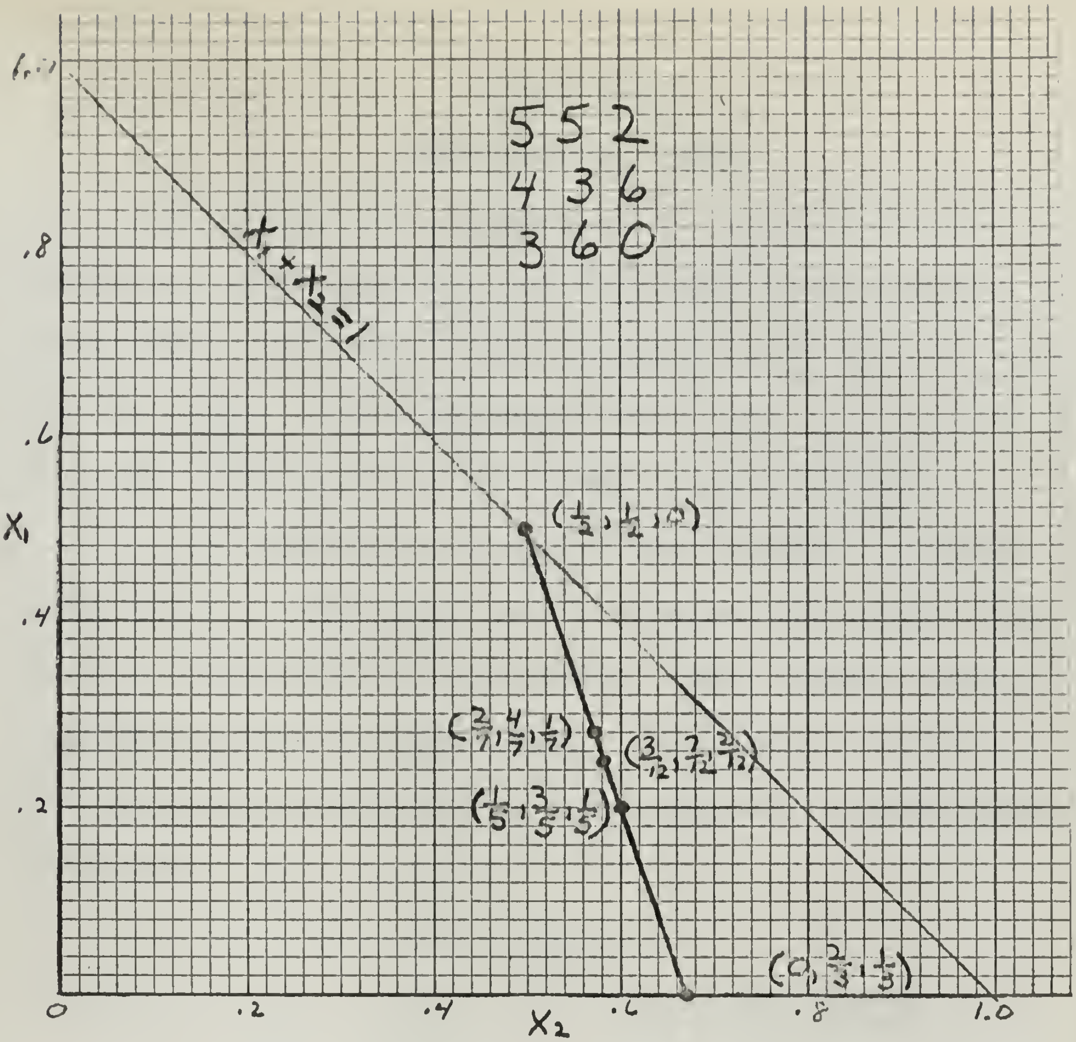
The two methods which are to be discussed next were successful. One is the improved method, as discussed in the previous chapter, as its optimal strategies at various iterations run up and down the line of optimal strategies. See Fig. 11, which is a plot of the optimal strategies contained

in the first 12 iterations of

$$\begin{vmatrix} 5 & 5 & 2 \\ 4 & 3 & 6 \\ 3 & 6 & 0 \end{vmatrix}$$

which is tabulated in A-4.

The final method attempted and which was also quite useful, but only in certain classes of games, is the complementary game method which is discussed in considerable detail in the following chapter.



$X_1 + X_2 + X_3 = 1$ Figure 11

From A-4: The following are the optimal strategies for Player I.

Play No.	Strategy	Between Play N and N'
1	$(\frac{1}{2}\ \frac{1}{2}\ 0)$	(1-2)
2	$(\frac{1}{2}\ \frac{1}{2}\ 0)$	(3-4)
3	$(\frac{1}{2}\ \frac{1}{2}\ 0)$	(1-4)
4	$(0\ \frac{2}{3}\ \frac{1}{3})$	(5-7)
5	$(\frac{2}{7}\ \frac{4}{7}\ \frac{1}{7})$	(1-7)
6	$(\frac{1}{5}\ \frac{3}{5}\ \frac{1}{5})$	(8-12)
7	$(\frac{3}{12}\ \frac{7}{12}\ \frac{2}{12})$	(1-12)

A plot of these values shows the complete line of solution for both the regular game $(\frac{1}{2},\frac{1}{2},0)$ to $(\frac{1}{5},\frac{3}{5},\frac{1}{5})$ and the complementary game $(\frac{1}{5},\frac{3}{5},\frac{1}{5})$ to $(0,\frac{2}{3},\frac{1}{3})$.

CHAPTER VII

COMPLEMENTARY GAME

The concept of a complementary game is useful in a game which has more than one solution and which also has the property that the value of the original game and its complement are the same. The complement of a game is the game formed from the transpose of the original game matrix.

It is possible to combine the optimal strategies of a game and its complement and determine the common point on a line of solutions of both games. This common point is a simple solution to both games. Two games, one a 3 x 3 and the other a 3 x 4, will be investigated in some detail to demonstrate this feature.

The first considered will be

$$A = \begin{Vmatrix} 2 & 5 & 5 \\ 6 & 3 & 4 \\ 0 & 6 & 3 \end{Vmatrix} \qquad \begin{aligned} X^*_1 &= (\tfrac{1}{2}, \tfrac{1}{2}, 0) \\ X^*_2 &= (\tfrac{1}{4}, 7/12, 1/6) \\ Y^* &= (1/3, 2/3, 0) \\ v &= 4 \end{aligned}$$

which is the game considered in some detail throughout most of this paper, and its complement

$$A' = \begin{Vmatrix} 2 & 6 & 0 \\ 5 & 3 & 6 \\ 5 & 4 & 3 \end{Vmatrix} \qquad \begin{aligned} X^*c &= (1/3, 2/3, 0) \\ Y^*c &= (1/8, 5/8, 2/8) \\ v &= 4 \end{aligned}$$

where X^*c is the optimal strategy of the complementary game. The values obtained for the solutions were the first solutions available by the improved method, as given in the Appendix (A-12 and A-39).

It is to be noted that Player I of the original game

[illegible]

becomes Player II in the complementary game.

X_1^*, X_2^*, and Y_c^* are collinear since the determinant

$$\begin{vmatrix} 1/2 & 1/2 & 0 \\ 1/4 & 7/12 & 1/6 \\ 1/8 & 5/8 & 1/4 \end{vmatrix} = 0$$

Hence, it is to be expected that some linear combination of X_1^* and Y_c^* will be a point of separation for the sets of optimal strategies for Player I in game A and for Player II in game A'. Consider $AX_1^* + (1-A)Y^*c$ and $BY^* + (1-B)X^*c$. An examination of Y^* and X_c^* shows they are identical and will, therefore, satisfy both the game and its complement. In the case of X_1^* it will not satisfy the complementary game, and neither will Y^*c act as an optimal strategy in the original game.

A check of the values of the columns in the original game yields:

$$\left\| \begin{matrix} 2 \times \frac{1}{2} + 6 \times \frac{1}{2} + 0 \times 0 \\ 5 \times \frac{1}{2} + 3 \times \frac{1}{2} + 0 \times 6 \\ 5 \times \frac{1}{2} + 4 \times \frac{1}{2} + 0 \times 3 \end{matrix} \right\| = \left\| \begin{matrix} 4 \\ 4 \\ 4\frac{1}{2} \end{matrix} \right\| \quad \text{or} \quad (X^{*\prime}A)' = \left\| \begin{matrix} 4 \\ 4 \\ 4\frac{1}{2} \end{matrix} \right\|$$

This is an optimal strategy for Player I in the original game. However, in the role of Player II in the complementary game, the third value of $4\frac{1}{2}$ would not be allowed in an optimal strategy.

Similarly, (1/8, 5/8, 2/8), which is an optimal strategy for Player II in the complementary game, is not allowable in the original game for Player I:

$$\begin{Vmatrix} 2 \times \frac{1}{8} + 6 \times \frac{5}{8} + 0 \times \frac{2}{8} \\ 5 \times \frac{1}{8} + 3 \times \frac{5}{8} + 6 \times \frac{2}{8} \\ 5 \times \frac{1}{8} + 4 \times \frac{5}{8} + 3 \times \frac{2}{8} \end{Vmatrix} = \begin{Vmatrix} 4 \\ 4 \\ 3\text{-}7/8 \end{Vmatrix} \quad \text{or} \quad AY^*_c = \begin{Vmatrix} 4 \\ 4 \\ 3\text{-}7/8 \end{Vmatrix}$$

Since all values here are equal to or less than 4, the value of the game, this satisfies the requirement for the minimizing player. Linearly combining these two "solutions" in the third column of the original game or third row of the complementary game, equating to the value of the game and then solving for A yields the following:

$$[A \times \tfrac{1}{2} + (1-A)\ 1/8]\ 5 + [A \times \tfrac{1}{2} + (1-A)\ 5/8]\ 4 + [A \times 0 + (1-A)\ 2/8]\ 3 = 4$$

or $A = 1/5$ so that the linear combination $1/5X^* + 4/5Y^*c = (1/5, 3/5, 1/5)$ results.

A check of this strategy in the original game (and also in the complement) shows that it gives exactly the value for each strategy of the opponent and is, therefore, a simple solution and an end point on the line of solutions for either game.

Just knowing one solution in the original game above, and one in its complementary game, is sufficient to determine the entire line of solutions. Going back to A-12 and picking the solution for Player I at Step 12 in the original game, we obtain $X^*_2 = (1/4, 7/12, 1/6)$. This point is used rather than the $(\frac{1}{2}\ \frac{1}{2}\ 0)$ as this point is not a basic optimal strategy, whereas the $(\frac{1}{2}\ \frac{1}{2}\ 0)$ is a basic optimal

[illegible]

strategy. The point $(1/8, 5/8, \frac{1}{4})$ optimal strategy from the complementary game will still be used. These two points will be combined linearly componentwise and equated to zero

$$\tfrac{1}{4}A + (1-A)(1/8) = 0$$

$$A = -1 \qquad (1-A) = 2$$

$$-1(\tfrac{1}{4}, 7/12, 1/6) + 2(1/8, 5/8, \tfrac{1}{4}) = (0, 2/3, 1/3)$$

This is a basic optimal strategy for the complementary game, but is not an optimal strategy for the original game

$$A(7/12) + (1-A)(5/8) = 0$$

$$A = 15 \qquad (1-A) = -14$$

This yields for an optimal strategy $(2, 0, -1)$ which, of course, is not allowed. Now, equating the third components gives

$$A(1/6) + (1-A)\tfrac{1}{2} = 0$$

$$A = 3 \qquad (1-A) = -2$$

Combining the two points in the above ratios gives $(\frac{1}{2}\ \frac{1}{2}\ 0)$, which is a basic solution to the original game. Therefore, when a line of solutions exists, as above, and the complementary game has the same value as the original game, it is possible to determine both the common point solution of both games and the end points of the line of solutions.

A 3 x 4 game will now be examined which satisfies the condition that the value of the game A is equal to the value of A', the complementary game. Therefore, the methods applicable to the 3 x 3 game discussed previously will also

apply here. It will be demonstrated that with only three points (optimal strategies), two from the original game and one from the complementary game are sufficient to obtain all optimal strategies (in this particular case, a plane).

$$A = \begin{Vmatrix} 0 & 1 & 2 & 3 \\ 3 & 2 & 1 & 0 \\ 0 & 2 & 1 & 3 \end{Vmatrix} \qquad A' = \begin{Vmatrix} 0 & 3 & 0 \\ 1 & 2 & 2 \\ 2 & 1 & 1 \\ 3 & 0 & 3 \end{Vmatrix}$$

$v = 3/2$ for both games

Two optimal strategies for Player II in A as determined by the improved method were $Y_1^* = (4/10, 0, 3/10, 3/10)$ and $Y_2^* = (1/3, 1/12, 1/3, 1/4)$, while the strategy for Player I in the complementary game was $X_c^* = (1/6, \frac{1}{2}, 0, 1/3)$. (More solutions than these were obtained from the run, but this is an attempt to extract all solutions from a minimum number.)

$$AY_1^* = \begin{Vmatrix} \frac{3}{2} \\ \frac{3}{2} \\ \frac{6}{5} \end{Vmatrix} \qquad AY_2^* = \begin{Vmatrix} \frac{3}{2} \\ \frac{3}{2} \\ \frac{5}{4} \end{Vmatrix} \qquad (X_c^*)'A' = \begin{Vmatrix} \frac{3}{2}, & \frac{3}{2}, & 2 \end{Vmatrix}$$

An examination of the results of the above products shows that the third row does not yield exactly the value in the original game. In the complementary game, the third column (which is the same as the third row in the original game) does not yield exactly the value of the game.

Solve the following two equations for parameters A and B to determine two points on the common line of solutions

to both the game and its complement:

$$1. \quad [A X_c^* + (1-A) Y^*] \begin{Vmatrix} 0 \\ 1 \\ 2 \\ 3 \end{Vmatrix} = \frac{3}{2}$$

$$2. \quad [B X_c^* + (1-B) Y^*] \begin{Vmatrix} 0 \\ 1 \\ 2 \\ 3 \end{Vmatrix} = \frac{3}{2}$$

$$A = \frac{3}{8}, \quad [A X^* + (1-A) Y^*] = \left(\frac{5}{16}, \frac{3}{16}, \frac{3}{16}, \frac{5}{16}\right)$$

$$B = \frac{1}{3}, \quad [B X^* + (1-B) Y^*] = \left(\frac{5}{18}, \frac{4}{18}, \frac{4}{18}, \frac{5}{18}\right)$$

Components will now be linearly combined to find the ends of the line which contains the points $\left(\frac{5}{16}, \frac{3}{16}, \frac{3}{16}, \frac{5}{16}\right)$ and $\left(\frac{5}{18}, \frac{4}{18}, \frac{4}{18}, \frac{5}{18}\right)$ and still satisfies the game.

$$C\left(\frac{5}{18}\right) + (1-C)\frac{5}{16} = 0 \qquad C = 9 \qquad (1-C) = -8$$

$$9\left(\frac{5}{18}, \frac{4}{18}, \frac{4}{18}, \frac{5}{18}\right) - 8\left(\frac{5}{16}, \frac{3}{16}, \frac{3}{16}, \frac{5}{16}\right) = \left(0, \tfrac{1}{2}, \tfrac{1}{2}, 0\right)$$

One end of the line is therefore $(0\ \tfrac{1}{2}\ \tfrac{1}{2}\ 0)$ which checks as an optimal strategy in both the original and complementary games and is a basic optimal strategy in both.

$$D\left(\frac{4}{18}\right) + (1-D)\frac{3}{16} = 0 \qquad D = \frac{-27}{5} \qquad (1-D) = \frac{32}{5}$$

$$\frac{-27}{5}\left(\frac{5}{18}\right) + \frac{32}{5}\left(\frac{5}{16}\right) = \tfrac{1}{2} \qquad \frac{-27}{5}\left(\frac{4}{18}\right) + \frac{32}{5}\left(\frac{3}{16}\right) = 0$$

The other end of the line and, again, a basic optimal strategy for both games is $(\tfrac{1}{2}\ 0\ 0\ \tfrac{1}{2})$.

Next, the points $\left(\frac{4}{10}, 0, \frac{3}{10}, \frac{3}{10}\right)$ and $(\tfrac{1}{2}\ 0\ 0\ \tfrac{1}{2})$ will be linearly combined componentwise to determine the end of their common line. $E\left(\frac{4}{10}\right) + (1-E)\left(\tfrac{1}{2}\right) = 0$

$$E = 5 \qquad (1-E) = -4$$

This yields the strategy $(0, 0, \frac{3}{2}, -\frac{1}{2})$ which is, of course, not allowed. Equating the fourth components, however, give the results:

$$E \frac{3}{10} + (1-E) \frac{1}{2} = 0$$

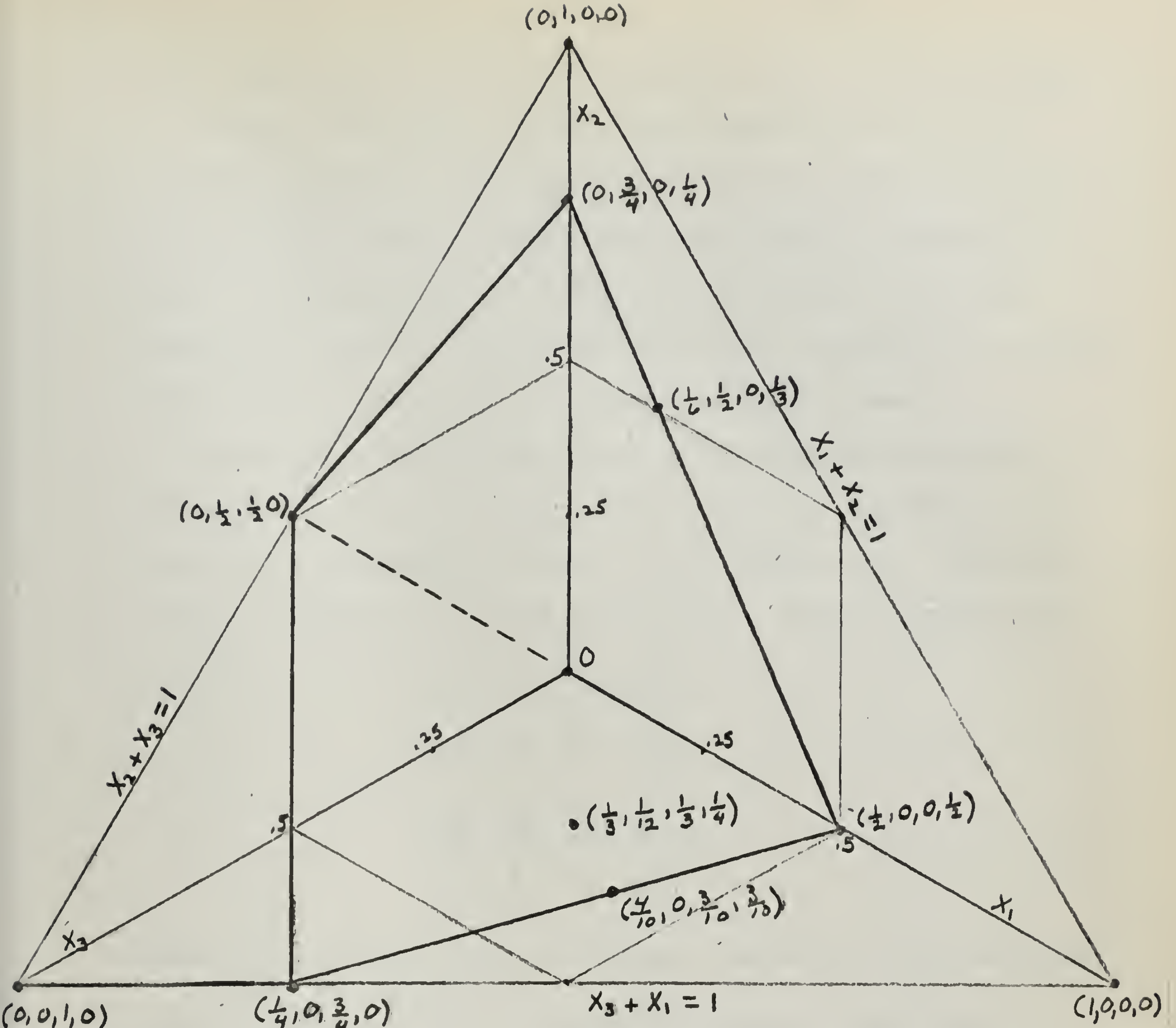
$$E = \frac{5}{2} \qquad (1-E) = -\frac{3}{2}$$

This yields $(\frac{1}{4}, 0, \frac{3}{4}, 0)$ when combining $\frac{5}{2} (\frac{4}{10}, 0, \frac{3}{10}, \frac{3}{10})$ with $-\frac{3}{2} (\frac{1}{2}, 0, 0, \frac{1}{2})$. This is again a basic optimal strategy for the original game; however, it is not a solution to the complementary game. This same solution could have been found by first combining $(\frac{1}{2}\ 0\ 0\ \frac{1}{2})$ and $(\frac{1}{3}, \frac{1}{12}, \frac{1}{3}, \frac{1}{4})$, then combining $(\frac{1}{6}, \frac{1}{6}, \frac{2}{3}, 0)$, the resultant, with $(0\ \frac{1}{2}\ \frac{1}{2}\ 0)$ to obtain the identical result of $(\frac{1}{4}, 0, \frac{3}{4}, 0)$.

To obtain the other basic strategy in the complementary game, it is necessary to combine $(\frac{1}{6}, \frac{1}{2}, 0, \frac{1}{3})$ with $(\frac{1}{2}\ 0\ 0\ \frac{1}{2})$; this then yields $(0, \frac{3}{4}, 0, \frac{1}{4})$ which is a solution to the complementary game, but not to the original game.

A three-dimensional graph of the plane of solutions is given in Fig. 12 for both the original game and its complement. It is to be noted that in form, the solutions of the complementary game are a mirror image of the solutions of the original game.

Although there is no proof available, it seems reasonable to conjecture that each time a game and its complement have the same value and a line or a plane of optimal strategies



The Optimal Strategy for a 3 x 4 Game and Its Complement

Figure 12

$X_1 + X_2 + X_3 + X_4 = 1$

The plane of solutions of the game $\begin{Vmatrix} 0 & 1 & 2 & 3 \\ 3 & 2 & 1 & 0 \\ 0 & 2 & 1 & 3 \end{Vmatrix}$

The points $(0,\frac{1}{2},\frac{1}{2},0)$, $(\frac{1}{4},0,3/4,0)$, $(\frac{1}{2},0,0,\frac{1}{2})$ determine the triangle bounded plane of optimal strategies for the above game. The points $(0,\frac{1}{2},\frac{1}{2},0)$, $(0,3/4,0,\frac{1}{4})$, $(0,\frac{1}{2},\frac{1}{2},0)$ determine the boundary for the optimal strategies of the complementary game.

exist; then this line or plane extends into a line or plane of optimal strategies in the complementary game and the dividing point, or dividing line between the two solutions, may be found. As is evidenced by the line of solutions in Fig. 11 and the plane of solutions in Fig. 12, the term complement seems to be fitting due to the symmetry of optimal strategies in the original and complementary game.

One could extend the notion of proving collinearity, as was shown in the 3 x 3 game, to a notion of coplanarity by forming the determinant which, if it equals zero, indicates that all points lie in the same plane. In the example considered

$$\begin{vmatrix} \frac{4}{10}, & 0, & \frac{3}{10}, & \frac{3}{10} \\ \frac{1}{3}, & \frac{1}{12}, & \frac{1}{3}, & \frac{1}{4} \\ \frac{5}{16}, & \frac{1}{16}, & \frac{7}{16}, & \frac{3}{16} \\ \frac{1}{6}, & \frac{1}{2}, & 0, & \frac{1}{3} \end{vmatrix} = 0$$

where $\left(\frac{5}{16}, \frac{1}{16}, \frac{7}{16}, \frac{3}{16}\right)$ was another solution of the original game as determined by the improved method. This check does require one more solution from the computer.

CHAPTER VIII

ANALYSIS OF A RANDOM GAME

This chapter gives an examination of a random 8 x 8 game. The elements of the matrix were picked from a table of random numbers, and the game was then solved on the computer using the improved Brown method.

$$\left\|\begin{matrix} 5 & 4 & 0 & 2 & 3 & 8 & 8 & 1 \\ 6 & 9 & 7 & 1 & 2 & 4 & 7 & 4 \\ 6 & 2 & 3 & 2 & 8 & 5 & 5 & 3 \\ 2 & 8 & 4 & 5 & 7 & 3 & 8 & 9 \\ 3 & 9 & 4 & 0 & 2 & 7 & 4 & 6 \\ 2 & 6 & 9 & 2 & 7 & 5 & 3 & 3 \\ 4 & 5 & 1 & 1 & 3 & 7 & 9 & 7 \\ 3 & 0 & 0 & 4 & 6 & 2 & 1 & 0 \end{matrix}\right\|$$

The game was a class III game with Player I having an optimal strategy of (1/305, 8/305, 120/305, 174/305, 0, 2/305, 0, 0). Player II's optimal strategy was found to be (77/195, 0, 23/195, 95/195, 0, 0, 0, 0).

The value of the game from below (Player I) was 3.685; the value from above was 3.693, which is a solution to almost three significant figures. The value from below came on the 305th iteration whereas the value from above came on the 195th play and was never improved upon through the 305th play, leaving a suspicion that the value from above is quite close to the actual value of the game.

For Player I, strategy 1 was used only on the first iteration and strategy 6 was used only twice, so both of these were dropped from the optimal strategy for Player I. Both will be tested a little later in this chapter against Player II's optimal strategy; this will indicate they were properly dropped. The optimal strategy for Player I is

now (0, .026, .397, .577, 0, 0, 0, 0); Player II still has (.395, 0, .118, .487, 0, 0, 0, 0) for his optimal strategy. A check of the pure strategies of Player I against the optimal strategy for Player II and the optimal strategy of Player I against each pure strategy of Player II is tabulated in the following table, which indicates that all of the active strategies do yield the value of the game and that none of the inactive strategies do.

Table of $\sum a_{ij} y_j$ for $i = 1, 2, \ldots 8$

i	$\underline{v}$
1	2.949
2	3.692*
3	3.681*
4	3.705*
5	1.657
6	2.826
7	2.585
8	3.133

Table of $\sum x_i a_{ij}$ for $j = 1, 2, \ldots 8$

j	$\bar{v}$
1	3.683*
2	5.644
3	3.698*
4	3.697*
5	7.267
6	3.820
7	6.783
8	6.588

*Indicates value of the game and, therefore, a pure strategy which can be used in the optimal strategy.

Table 2

CHAPTER IX

FLOW CHART AND NCR-102A PROGRAM

The program flow chart is shown in two sections (Fig. 13 and Fig. 14): the first is the original Brown method; the second flow chart is an addition to the Brown method which eliminates the portion of the first chart, which is enclosed in dotted lines. The second flow diagram fits in between the point A1 and A2 on the original chart.

Memory cells 0000 through 0561 are used in the program proper, while the floating point sub-routine is inserted in 1000 and is about 100 octal words in length. The convert to floating point form sub-routine is put into 1700 and uses 21 octal words.

To utilize the routine it is required to put the game matrix up to 8 x 8 in size in cells 0000-0100, putting the first row into 0000-0007, the second row into 0010-0017, the third row into 0020-0027, etc. The number of rows should be entered in 0331, while the number of columns should be entered in 0330. The number of rows minus one is put into 0334, while the number of columns minus one goes into 0333. The value of $\bar{V}-\underline{V}$ desired for convergence is entered in 0332; it is entered as the exponential value in floating point form, as the $\bar{v}-\underline{v}$ generated by the computer for comparison is in floating point form.

The original Brown method program is entered from 0400-0561, while the modification extends from 0110-0172. All four test switches are in the program with the following

uses:

2100 when on prints the $\bar{v}$(hold)-$\underline{v}$(hold) in floating point form

2010 when on eliminates print of hold column tally and $\bar{v}$(hold)

2020 when on eliminates print of the hold row tally and $\underline{v}$(hold)

2040 when on prints $\underline{v}$, $\bar{v}$, $\bar{v}$-$\underline{v}$ in floating point form and N (iteration number) in octal

The following cells contain the following information during the operation of the program:

0100-0107	contains row sums
0200-0207	contains column sums
0300-0307	contains row tally
0310-0317	contains column tally
0320-0321	$\underline{v}$ F.P. form
0321-0322	$\bar{v}$ F.P. form
0324	iteration tally
0325-0326	$\bar{v}$-$\underline{v}$ F.P. form
0230-0237	row hold tally (Player I)
0240-0247	column hold tally (Player II)
0250-0251	$\underline{v}$(hold)F.P. form
0252	iteration (hold) from below
0253-0254	$\bar{v}$(hold)F.P. form
0255	iteration (hold) from above
0256-0257	$\bar{v}$(hold)-$\underline{v}$(hold)F.P. form

A one should be entered in 0300 (or remembered) for the first tally since, normally, the first row is picked as the

first selection. In case it is desired to select a different row to start the game, say the fourth row, then a one should be entered in 0304 instead of 0300; and the first step of the program (0400) should be altered to 35 000401040104.

The time per iteration is eighteen seconds for an 8 x 8 matrix. All entries in the game matrix should be non-negative integers.

The actual program is listed in Appendix B.

A step-by-step example of the Brown method is given in Chapter II. A step-by-step example of the modified form will be given here for the game

$$\begin{matrix} 2 & 5 & 5 \\ 6 & 4 & 3 \\ 0 & 3 & 6 \end{matrix}$$

which is graphed in Fig. 3 and has the Brown play in A-11. The following is a tabulation of computer prints with all test switches off:

Play	Computer Prints					
1	230 1	231 0	232 0	250-251	2(fp)	0252 1
	240 1	241 0	242 0	253-254	6(fp)	0255 1
2	230 1	231 1	232 0	250-251	4(fp)	0252 2
	240 2	241 0	242 0	253-254	6(fp)	0255 2
3	240 2	241 0	242 1	253-254	5(fp)	0255 3
4	240 2	241 0	242 2	253-254	4.5(fp)	255 4
5	240 2	241 0	242 3	253-254	4.2(fp)	255 5
6	240 2	241 0	242 4	253-254	4(fp)	255 6
	256-257 0(fp)					
	230 1 231 1 232 0 240 2 241 0 242 4 250-251 4(fp)					
	252 2 253-254 4(fp) 255 6					
7, 8,	no print					

[illegible]

[illegible]

[illegible]

[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

[illegible]

9 240 3 241 0 242 6 253-254 4(fp) 255 9 256-257 0(fp)
230 1 231 1 232 0 240 3 241 0 242 6
250-251 4(fp) 252 2 253-254 4(fp) 255 9

10, 11, no print

12 230 3 231 7 232 2 250-251 4(fp) 252 12 256-257 0(fp)
230 3 231 7 232 2 240 3 241 0 242 6
250-251 4(fp 252 12 253-254 4(fp) 255 9

For an explanation of the print out, examine play 2: The symbols 230 1 231 1 232 0 mean that Player I has picked row 1 (230) once, row 2 (231) once, and row 3 (232) zero times. The $\underline{v}$(hold) (250-251) is 4 and is in floating point form with cell 250 containing the exponential portion and 251 containing the fractional portion of the value. The iteration when this particular strategy and value occured is at play 2, indicated by 252 2. Player I's strategy through two plays is therefore $(\frac{1}{2}, \frac{1}{2}, 0)$.

The 240-242 cells contain Player II's strategy, while v(hold) is in 253-254 in floating point form, and the iteration when Player II's "hold" strategy occurred is in 255. Whenever 256-257 prints out, it contains v(hold) - $\underline{v}$(hold) for the strategies being held in 230-232 and 240-242.

If switches 2010 and 2020 had been on, only the print outs for plays 6, 9, 12 would have appeared and only the parts from 256-257 on in each case. An examination of play 6 shows that since $\overline{v}$(hold) - $\underline{v}$(hold) is 0 (256-257), the strategies are exact and $(\frac{1}{2}\ \frac{1}{2}\ 0)$ is the strategy for Player I and $\left(\frac{1}{3}, 0, \frac{2}{3}\right)$ is exact for Player II with value of the game being four. Play 9 gives exactly the same optimal strategies for both players as play 6; however, play 12 gives a new optimal

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

strategy for Player I of $(\frac{1}{4}, \frac{7}{12}, \frac{1}{6})$ while Player II still maintains his old strategy of $(\frac{1}{3}, 0, \frac{2}{3})$. A line of solutions therefore exists for Player I; the methods on page 36 can be used to determine the entire line. It is to be noted that on play 21 of this game (A-11), Player II picks up a strategy which is not used in the optimal mixture and from this play on, Player II will never have an exact strategy in a finite number of plays.

FLOW CHART FOR
BROWN METHOD

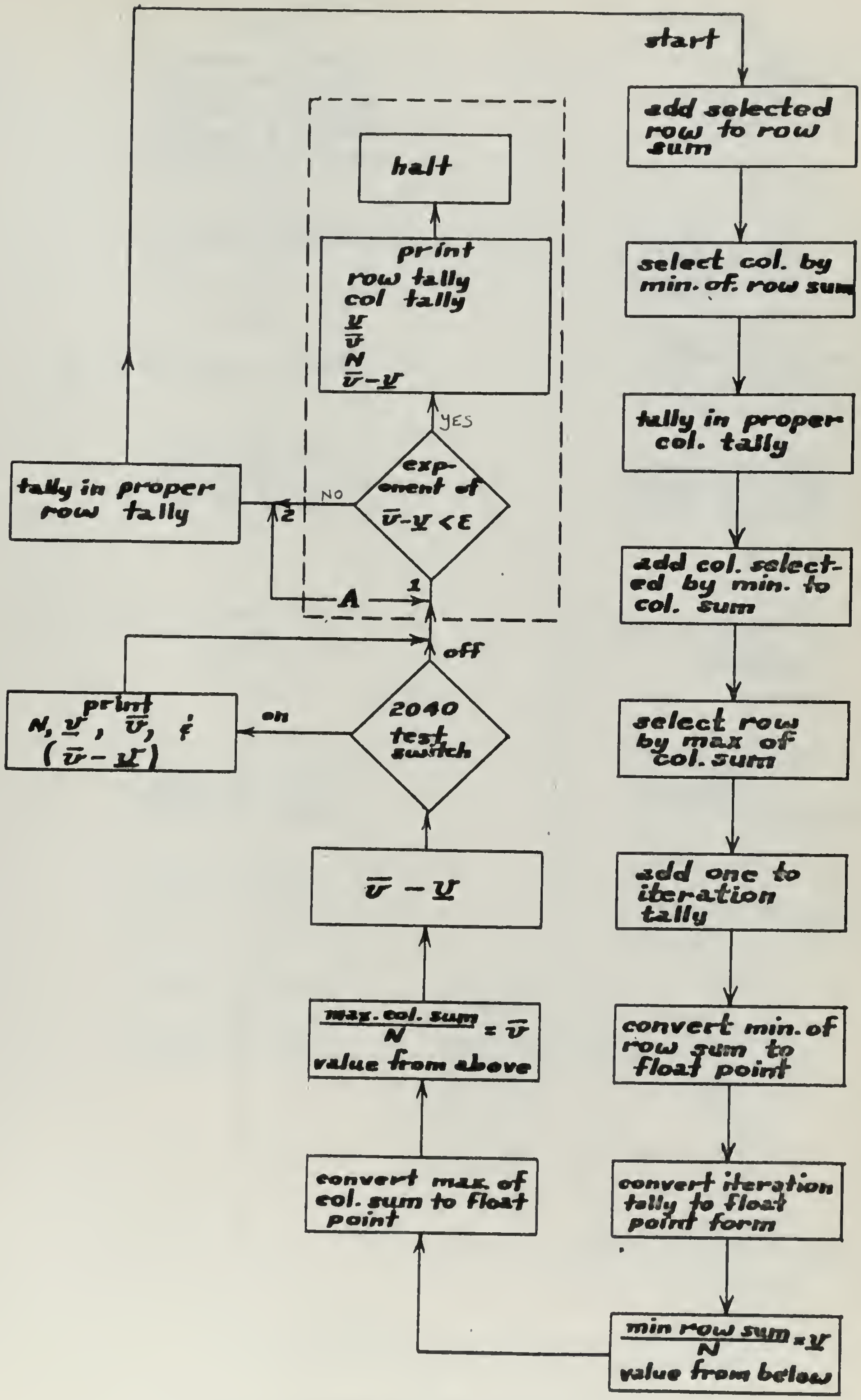
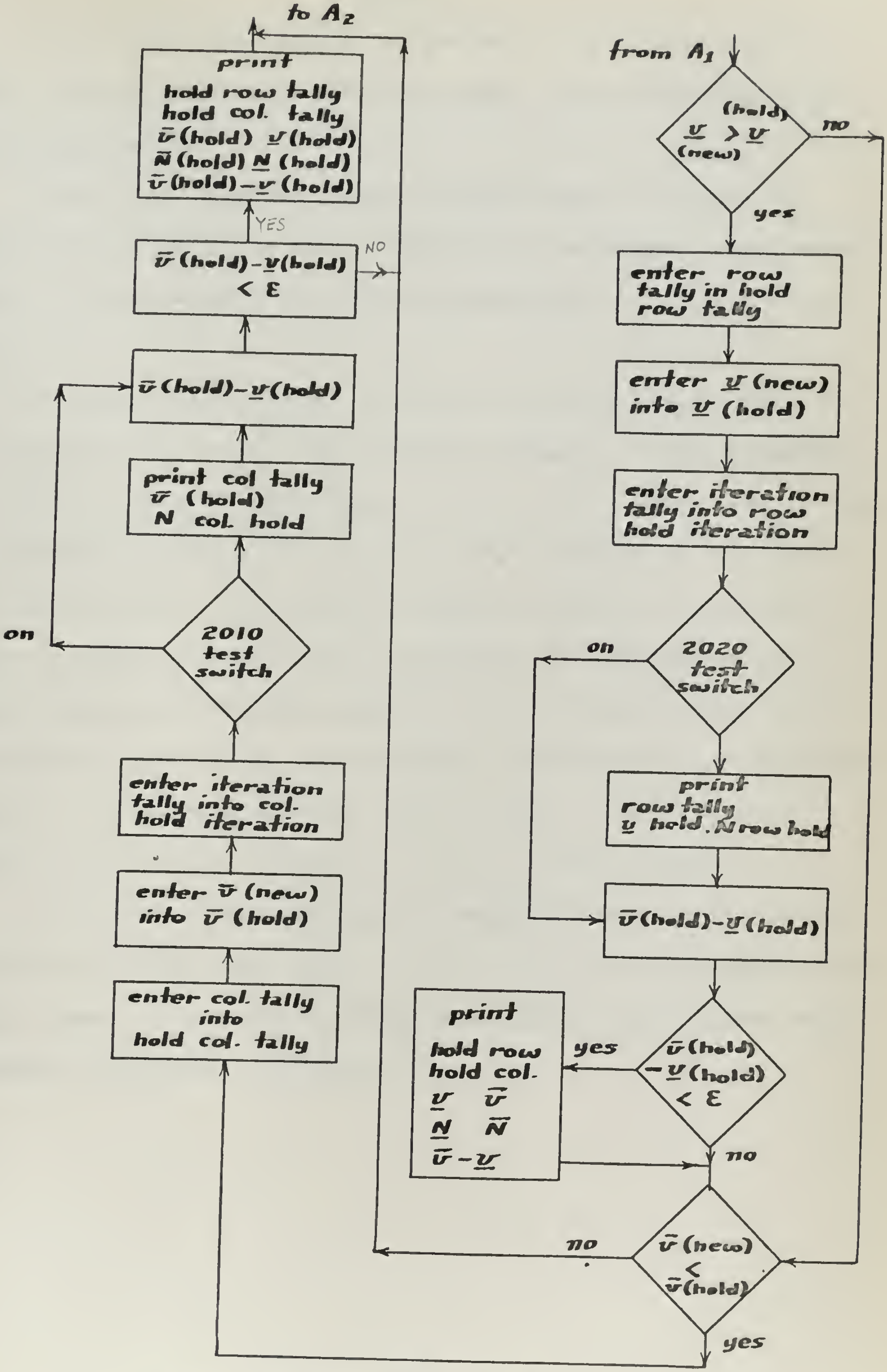


Figure 13

MODIFICATION TO BROWN METHOD



Insert between A_1 and A_2 on Flow Chart and eliminate the dotted portion of Brown Method

Figure 14

SUMMARY

It is felt that the improvement to the Brown method makes this a practical method to use, even with relatively slow computers.

There are, however, many problems left to solve to make this method even more effective--the primary one being, what is the optimal row column arrangement for speedy solution.

A second idea worthy of more investigation is that in a changing situation, the relative weights of the elements in the game matrix may change. It is often important in such a changing situation to also keep the effects of the past in the problem. This method should be ideal to partially solve a game with one set of elements in the game matrix, then substitute the new elements in at a proper time into the matrix and finish the solution to any degree of accuracy desired. The problem here would be when in the play to substitute the new elements to give a proper answer.

A further investigation of complementary games, including those where the value is not equal to the original game, might lead to some interesting and useful conclusions about optimal strategies in general.

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and

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APPENDIX

APPENDIX A

This appendix contains the first 30 iterations of all possible permutations of the rows and columns of the game matrix

$$\begin{Vmatrix} 5 & 5 & 2 \\ 4 & 3 & 6 \\ 3 & 6 & 0 \end{Vmatrix}$$

The only exceptions are the first 6 games (A-1 through A-6)--these have exact Brown solutions in 12 iterations and, therefore, play stops at the end of 12 plays. This appendix also contains 3 other game matricies which are used in the thesis--these are the last 3 games in the appendix.

The first 4 plays of A-4 are worked out below to insure understanding of the tables.

$$\begin{array}{cccc} & \boxed{1} & \boxed{2} & \boxed{3} \\ 1 & 5 & 5 & 2 \\ 2 & 4 & 3 & 6 \\ 3 & 3 & 6 & 0 \end{array}$$

		ROW SUM					COL. SUM				
N	i	T_1	T_2	T_3	$\underline{v}$	j	T_1	T_2	T_3	$\bar{v}$	$\bar{v} - \underline{v}$
1	1	5	5	2	2.	3	2	6	0	6.	4.
2	2	9	8	8	4.	2	7	9	6	4.5	.5
3	2	13	11	14	3.75	2	12	12	12	4.	.25
4	1	18	16	16	4.	2	17	15	18	4.5	.5

N is the number of the play.

i is the row played at play N by Player I.

j is the column played at play N by Player II.

T_1 T_2 T_3, adjacent to i are the row sum for Player I.

T_1 T_2 T_3, adjacent to j are the column sum for Player II.

[illegible]

$\underline{v}$ is the min of the row sum divided by N .

$\overline{v}$ is the max of the column sum divided by N .

$\overline{v} - \underline{v}$ is the difference between the current approximate value and the true value of the game.

As an example, at play 2, Player I has a strategy of row 1 once; row 2 once; and row 3 zero times, or $(\frac{1}{2}\frac{1}{2}0)$. His expectation, when playing this strategy, is at least $\underline{v}$ -- in this case, 4. Player II at play 2 has a strategy of $(0\frac{1}{2}\frac{1}{2})$ and his expectation--the most he can lose--is $\overline{v}$ (in this case, 4.5). This leaves $\overline{v} - \underline{v}$ (.5) in doubt. At play 3, the strategies are (1/3, 2/3, 0) for Player I; and his expectation has dropped to 3.75, while Player II has a strategy of (0, 2/3, 1/3) and his expectation has improved to -4. When $\overline{v} - \underline{v} = 0$, a solution by the Brown method has been generated.

	1	2	3
1	5	5	2
2	6	3	0
3	3	4	6

		ROW SUM					COL. SUM				
N	i	T_1	T_2	T_3	$\underline{v}$	j	T_1	T_2	T_3	$\bar{v}$	$\bar{v} - \underline{v}$
1	1	5	5	2	2.	3	2	0	6	6.	4.
2	3	8	9	8	4.	1	7	6	9	4.5	.5
3	3	11	13	14	3.66	1	12	12	12	4.	.34
4	1	16	18	16	4.	1	17	18	15	4.5	.5
5	2	22	21	16	3.2	3	19	18	21	4.2	1.0
6	3	25	25	22	3.66	3	21	18	27	4.5	.84
7	3	28	29	28	4.	1	26	24	30	4.29	.29
8	3	31	33	34	3.87	1	31	30	33	4.12	.25
9	3	34	37	40	3.77	1	36	36	36	4.0	.23
10	1	39	42	42	3.9	1	41	42	39	4.2	.3
11	2	45	45	42	3.81	3	43	42	45	4.09	.28
12	3	48	49	48	4.0	1	48	48	48	4.0	0

A-2

	1	2	3
1	5	2	5
2	3	6	4
3	6	0	3

		ROW SUM					COL. SUM				
N	i	T_1	T_2	T_3	$\underline{v}$	j	T_1	T_2	T_3	$\bar{v}$	$\bar{v} - \underline{v}$
1	1	5	2	5	2.	2	2	6	0	6.	4.
2	2	8	8	9	4.	1	7	9	6	4.5	.5
3	2	11	14	13	3.66	1	12	12	12	4.	.34
4	1	16	16	18	4.	1	17	15	18	4.5	.5
5	3	22	16	21	3.2	2	19	21	18	4.2	1.0
6	2	25	22	25	3.66	2	21	27	18	4.5	.84
7	2	28	28	29	4.	1	26	30	24	4.29	.29
8	2	31	34	33	3.87	1	31	33	30	4.12	.25
9	2	34	40	37	3.77	1	36	36	36	4.0	.23
10	1	39	42	42	3.9	1	41	39	42	4.2	.3
11	3	45	42	45	3.81	2	43	45	42	4.09	.28
12	2	48	48	49	4.0	1	48	48	48	4.0	0

A-3

	1	2	3
1	5	2	5
2	6	0	3
3	3	6	4

		ROW SUM					COL. SUM				
N	i	T_1	T_2	T_3	$\underline{v}$	j	T_1	T_2	T_3	$\bar{v}$	$\bar{v} - \underline{v}$
1	1	5	2	5	2.	2	2	0	6	6.	4.
2	3	8	8	9	4.0	1	7	6	9	4.5	.5
3	3	11	14	13	3.67	1	12	12	12	4.0	.33
4	1	16	16	18	4.0	1	17	18	15	4.5	.5
5	2	22	16	21	3.2	2	19	18	21	4.2	1.0
6	3	25	22	25	3.67	2	21	18	27	4.5	.88
7	3	28	28	29	4.0	1	26	24	30	4.29	.29
8	3	31	34	33	3.87	1	31	30	33	4.12	.25
9	3	34	40	37	3.77	1	36	36	36	4.0	.23
10	1	39	42	42	3.9	1	41	42	39	4.2	.3
11	2	45	42	45	3.81	2	43	42	45	4.09	.28
12	3	48	48	49	4.0	1	48	48	48	4.0	0

A-4

	1	2	3
1	5	5	2
2	4	3	6
3	3	6	0

		ROW SUM					COL. SUM				
N	i	T_1	T_2	T_3	$\underline{v}$	j	T_1	T_2	T_3	$\bar{v}$	$\bar{v} - \underline{v}$
1	1	5	5	2	2.	3	2	6	0	6.	4.
2	2	9	8	4	4.	2	7	9	6	4.5	.5
3	2	13	11	14	3.67	2	12	12	12	4.0	.33
4	1	18	16	16	4.0	2	17	15	18	4.5	.5
5	3	21	22	16	3.2	3	19	21	18	4.2	1.
6	2	25	25	22	3.67	3	21	27	18	4.5	.83
7	2	29	28	28	4.	2	26	30	24	4.28	.28
8	2	33	31	34	3.87	2	31	33	30	4.13	.25
9	2	37	34	40	3.76	2	36	36	36	4.0	.24
10	1	42	39	42	3.9	2	41	39	42	4.2	.3
11	3	45	45	42	3.8	3	43	45	42	4.09	.29
12	2	49	48	48	4.	2	48	48	48	4.0	0

	1	2	3
1	5	5	2
2	3	6	0
3	4	3	6

		ROW SUM					COL. SUM				
N	i	T_1	T_2	T_3	$\underline{v}$	j	T_1	T_2	T_3	$\bar{v}$	$\bar{v} - \underline{v}$
1	1	5	5	2	2.	3	2	0	6	6.	4.
2	3	9	8	8	4.	2	7	6	9	9.5	.5
3	3	13	11	14	3.67	2	12	12	12	4.	.33
4	1	18	16	16	4.	2	17	18	15	4.5	.5
5	2	21	22	16	3.2	3	19	18	21	4.2	1.
6	3	25	25	22	3.67	3	21	18	27	4.5	.83
7	3	29	28	28	4.	2	26	24	30	4.28	.28
8	3	33	31	34	3.87	2	31	30	33	4.12	.25
9	3	37	34	40	3.77	2	36	36	36	4.	.23
10	1	42	39	42	3.9	2	41	42	39	4.2	.3
11	2	45	45	42	3.82	3	43	42	45	4.09	.27
12	3	49	48	48	4.0	2	48	48	48	4.	0

A-6

	1	2	3
1	5	5	2
2	3	4	6
3	6	3	0

		ROW SUM					COL. SUM				
N	i	T_1	T_2	T_3	$\underline{v}$	j	T_1	T_2	T_3	$\bar{v}$	$\bar{v} - \underline{v}$
1	1	5	5	2	2.	3	2	6	0	6.	4.
2	2	8	9	8	4.	1	7	9	6	4.5	.5
3	2	11	13	14	3.67	1	12	12	12	4.	.33
4	1	16	18	16	4.	1	17	15	18	4.5	.5
5	3	22	21	16	3.2	3	19	21	18	4.2	1.
6	2	25	25	22	3.67	3	21	27	18	4.5	.83
7	2	28	29	28	4.	1	26	30	24	4.28	.28
8	2	31	33	34	3.87	1	31	33	30	4.12	.25
9	2	34	37	40	3.77	1	36	36	36	4.	.23
10	1	39	42	42	3.9	1	41	39	42	4.2	.3
11	3	45	45	42	3.82	3	43	45	42	4.09	.27
12	2	48	49	48	4.0	1	48	48	48	4.0	0

	1	2	3
1	6	0	3
2	5	2	5
3	3	6	4

		ROW SUM					COL. SUM				
N	i	T_1	T_2	T_3	$\underline{v}$	j	T_1	T_2	T_3	$\bar{v}$	$\bar{v} - \underline{v}$
1	1	6	0	3	0	2	0	2	6	6.	6.
2	3	9	6	7	3.	2	0	4	12	6.	3.
3	3	12	12	11	3.67	1	6	9	15	5.	1.33
4	3	15	18	15	3.75	1	12	14	18	4.5	.75
5	3	18	24	19	3.6	1	18	19	21	4.2	.6
6	3	21	30	23	3.5	1	24	24	24	4.0	.5
7	1	27	30	26	3.72	3	27	29	28	4.14	.42
8	2	32	32	31	3.87	3	30	34	32	4.25	.37
9	2	37	34	36	3.78	2	30	36	38	4.22	.44
10	3	40	40	40	4.0	1	36	41	41	4.1	.1
11	2	45	42	45	3.81	2	36	43	47	4.27	.46
12	3	48	48	49	4.0	1	42	48	50	4.17	.17
13	3	51	54	53	3.92	1	48	53	53	4.07	.15
14	2	56	56	58	4.0	1	54	58	56	4.14	.14
15	2	61	58	63	3.87	2	54	60	62	4.13	.26
16	3	64	64	67	4.0	1	60	65	65	4.06	.06
17	2	69	66	72	3.89	2	60	67	71	4.17	.28
18	3	72	72	76	4.0	1	66	72	74	4.11	.11
19	3	75	78	80	3.95	1	72	77	77	4.05	.1
20	2	80	80	85	4.0	1	78	82	80	4.1	.1
21	2	85	82	90	3.9	2	78	84	86	4.09	.19
22	3	88	88	94	4.0	1	84	89	89	4.04	.04
23	2	93	90	99	3.91	2	84	91	95	4.13	.22
24	3	96	96	103	4.0	1	90	96	98	4.08	.08
25	3	99	102	107	3.96	1	96	101	101	4.03	.07
26	1	105	102	110	3.93	2	96	103	107	4.12	.19
27	3	108	108	114	4.0	1	102	108	110	4.07	.07
28	3	111	114	118	3.96	1	108	113	113	4.03	.07
29	2	116	116	123	4.0	1	114	118	116	4.07	.07
30	2	121	118	128	3.93	2	114	120	122	4.07	.14

	1	2	3
1	2	5	5
2	0	6	3
3	6	3	4

		ROW SUM					COL. SUM				
N	i	T_1	T_2	T_3	$\underline{v}$	j	T_1	T_2	T_3	$\overline{v}$	$\overline{v} - \underline{v}$
1	1	2	5	5	2.	1	2	0	6	6.	4.
2	3	8	8	9	4.	1	4	0	12	6.	2.
3	3	14	11	13	3.67	2	9	6	15	5.	1.33
4	3	20	14	17	3.5	2	14	12	18	4.5	1.
5	3	26	17	21	3.4	2	19	18	21	4.2	.8
6	3	32	20	25	3.33	2	24	24	24	4.0	.67
7	1	34	25	30	3.57	2	29	30	27	4.28	.71
8	2	34	31	33	3.87	2	34	36	30	4.5	.63
9	2	34	37	36	3.77	1	36	36	36	4.0	.23
10	1	36	42	41	3.6	1	38	36	42	4.2	.6
11	3	42	45	45	3.81	1	40	36	48	4.37	.56
12	3	48	48	49	4.0	1	42	36	54	4.5	.5
13	3	54	51	53	3.92	2	47	42	57	4.38	.46
14	3	60	54	57	3.85	2	52	48	60	4.28	.43
15	3	66	57	61	3.8	2	57	54	63	4.2	.4
16	3	72	60	65	3.75	2	62	60	66	4.12	.37
17	3	78	63	69	3.71	2	67	66	69	4.06	.35
18	3	84	66	73	3.67	2	72	72	72	4.0	.33
19	1	86	71	78	3.73	2	77	78	75	4.11	.38
20	2	86	77	81	3.85	2	82	84	78	4.2	.35
21	2	86	83	84	3.95	2	87	90	81	4.28	.33
22	2	86	89	87	3.91	1	89	90	87	4.09	.18
23	2	86	95	90	3.74	1	91	90	93	4.04	.30
24	3	92	98	94	3.83	1	93	90	99	4.12	.29
25	3	98	101	98	3.92	1	95	90	105	4.2	.28
26	3	104	104	102	3.93	3	100	93	109	4.19	.19
27	3	110	107	106	3.94	3	105	96	113	4.18	.24
28	3	116	110	110	3.93	2	110	102	116	4.15	.22
29	3	122	113	114	3.9	2	115	108	119	4.11	.21
30	3	128	116	118	3.87	2	120	114	122	4.06	.19

$$\begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & 2 & 5 & 5 \\ 2 & 0 & 3 & 6 \\ 3 & 6 & 4 & 3 \end{array}$$

		ROW SUM					COL. SUM				
N	i	T_1	T_2	T_3	$\underline{v}$	j	T_1	T_2	T_3	$\bar{v}$	$\bar{v} - \underline{v}$
1	1	2	5	5	2.	1	2	0	6	6.	4.
2	3	8	9	8	4.	1	4	0	12	6.	2.
3	3	14	13	11	3.75	3	9	6	15	5.	1.25
4	3	20	17	14	3.5	3	14	12	18	4.5	1.
5	3	26	21	17	3.4	3	19	18	21	4.2	.8
6	3	32	25	20	3.33	3	24	24	24	4.	.67
7	1	34	30	25	3.43	3	29	30	27	4.29	.86
8	2	34	33	31	3.87	3	34	36	30	4.5	.63
9	2	34	36	37	3.77	1	36	36	36	4.0	.23
10	1	36	41	42	3.6	1	38	36	42	4.2	.6
11	3	42	45	45	3.82	1	40	36	48	4.36	.54
12	3	48	49	48	4.	1	42	36	54	4.5	.5
13	3	54	53	51	3.92	3	47	42	57	4.38	.46
14	3	60	57	54	3.86	3	52	48	60	4.28	.42
15	3	66	61	57	3.8	3	57	54	63	4.20	.4
16	3	72	65	60	3.74	3	62	60	66	4.13	.39
17	3	78	69	63	3.71	3	67	66	69	4.06	.35
18	3	84	73	66	3.66	3	72	72	72	4.	.34
19	1	86	78	71	3.73	3	77	78	75	4.11	.38
20	2	86	81	77	3.84	3	82	84	78	4.2	.36
21	2	86	84	83	3.95	3	87	90	81	4.28	.33
22	2	86	87	89	3.9	1	89	90	87	4.22	.32
23	2	86	90	95	3.73	1	91	90	93	4.04	.31
24	3	92	94	98	3.83	1	93	90	99	4.11	.28
25	3	98	98	101	3.92	1	95	90	105	4.2	.28
26	3	104	102	104	3.93	2	100	93	109	4.19	.26
27	3	110	106	107	3.96	2	105	96	113	4.18	.22
28	3	116	110	110	3.93	2	110	99	117	4.17	.24
29	3	122	114	113	3.89	3	115	105	120	4.14	.25
30	3	128	118	116	3.86	3	120	111	123	4.1	.24

	1	2	3
1	5	2	5
2	3	0	6
3	4	6	3

		ROW SUM					COL. SUM				
N	i	T_1	T_2	T_3	$\underline{v}$	j	T_1	T_2	T_3	$\bar{v}$	$\bar{v} - \underline{v}$
1	1	5	2	5	2.	2	2	0	6	6.	4.
2	3	9	8	8	4.	2	4	0	12	6.	2.
3	3	13	14	11	3.67	3	9	6	15	5.	1.33
4	3	17	20	14	3.5	3	14	12	18	4.5	1.
5	3	21	26	17	3.4	3	19	18	21	4.2	.8
6	3	25	32	20	3.33	3	24	24	24	4.	.67
7	1	30	34	25	3.57	3	29	30	27	4.29	.62
8	2	33	34	31	3.87	3	34	36	30	4.5	.63
9	2	36	34	37	3.77	2	36	36	36	4.	.23
10	1	41	36	42	3.6	2	38	36	42	4.2	.6
11	3	45	42	45	3.81	2	40	36	48	4.36	.55
12	3	49	48	48	4.	2	42	36	54	4.5	.5
13	3	53	54	51	3.92	3	47	42	57	4.38	.46
14	3	57	60	54	3.85	3	52	48	60	4.28	.43
15	3	61	66	57	3.8	3	57	54	63	4.2	.4
16	3	65	72	60	3.75	3	62	60	66	4.12	.37
17	2	68	72	66	3.89	3	67	66	69	4.06	.17
18	3	72	78	69	3.83	3	72	72	72	4.	.17
19	1	77	80	74	3.9	3	77	78	75	4.1	.2
20	2	79	80	80	3.95	2	79	78	81	4.05	.1
21	3	83	86	83	3.95	1	84	81	85	4.04	.09
22	3	87	92	86	3.91	3	89	87	88	4.04	.13
23	1	92	94	91	3.95	3	94	93	91	4.08	.13
24	1	97	96	96	4.	2	96	93	97	4.03	.03
25	3	101	102	99	3.96	3	101	99	100	4.04	.08
26	1	106	104	104	4.	2	103	99	106	4.07	.07
27	3	110	110	107	3.97	3	108	105	109	4.03	.06
28	3	114	116	110	3.93	3	113	111	112	4.03	.10
29	1	119	118	115	3.97	3	118	117	115	4.07	.10
30	1	124	120	120	4.	2	120	117	121	4.03	.03

	1	2	3
1	2	5	5
2	6	4	3
3	0	3	6

	ROW SUM					COL. SUM					
N	i	T1	T2	T3	$\underline{v}$	j	T1	T2	T3	$\bar{v}$	$\bar{v} - \underline{v}$
1	1	2	5	5	2.	1	2	6	0	6.	4.
2	2	8	9	8	4.	1	4	12	0	6.	2.
3	2	14	13	11	3.67	3	9	15	6	5.	1.33
4	2	20	17	14	3.5	3	14	18	12	4.5	1.
5	2	26	21	17	3.4	3	19	21	18	4.2	.8
6	2	32	25	20	3.33	3	24	24	24	4.	.67
7	1	34	30	25	3.57	3	29	27	30	4.28	.71
8	3	34	33	31	3.87	3	34	30	36	4.5	.63
9	3	34	36	37	3.77	1	36	36	36	4.	.23
10	1	36	41	42	3.6	1	38	42	36	4.2	.6
11	2	42	45	45	3.81	1	40	48	36	4.36	.55
12	2	48	49	48	4.	1	42	54	36	4.5	.5
13	2	54	53	51	3.92	3	47	57	42	4.38	.46
14	2	60	57	54	3.85	3	52	60	48	4.28	.43
15	2	66	61	57	3.8	3	57	63	54	4.2	.4
16	2	72	65	60	3.75	3	62	66	60	4.12	.37
17	2	78	69	63	3.71	3	67	69	66	4.06	.25
18	2	84	73	66	3.67	3	72	72	72	4.	.33
19	1	86	78	71	3.73	3	77	75	78	4.1	.37
20	3	86	81	77	3.85	3	82	78	84	4.2	.35
21	3	86	84	83	3.95	2	87	82	87	4.14	.19
22	1	88	89	88	4.	1	89	88	87	4.04	.04
23	1	90	94	93	3.91	1	91	94	87	4.08	.17
24	2	96	98	96	4.	1	93	100	87	4.16	.16
25	2	102	102	99	3.96	3	98	103	93	4.12	.16
26	2	108	106	102	3.93	3	103	106	99	4.08	.15
27	2	114	110	105	3.89	3	108	109	105	4.03	.14
28	2	120	114	108	3.85	3	113	112	111	4.03	.18
29	1	122	119	113	3.89	3	118	115	117	4.07	.18
30	1	124	124	118	3.93	3	123	118	123	4.1	.17

	1	2	3
1	2	5	5
2	6	3	4
3	0	6	3

		ROW SUM					COL. SUM				
N	i	T_1	T_2	T_3	$\underline{v}$	j	T_1	T_2	T_3	$\bar{v}$	$\bar{v} - \underline{v}$
1	1	2	5	5	2.	1	2	6	0	6.	4.
2	2	8	8	9	4.	1	4	12	0	6.	2.
3	2	14	11	13	3.67	2	9	15	6	5.	1.33
4	2	20	14	17	3.5	2	14	18	12	4.5	1.
5	2	26	17	21	3.4	2	19	21	18	4.2	.8
6	2	32	20	25	3.33	2	24	24	24	4.	.67
7	1	34	25	30	3.57	2	29	27	30	4.28	.71
8	3	34	31	33	3.88	2	34	30	36	4.5	.62
9	3	34	37	36	3.64	1	36	36	36	4.	.36
10	1	36	42	41	3.6	1	38	42	36	4.2	.6
11	2	42	45	45	3.8	1	40	48	36	4.37	.57
12	2	48	48	49	4.	1	42	54	36	4.5	.5
13	2	54	51	53	3.92	2	47	57	42	4.38	.44
14	2	60	54	57	3.86	2	52	60	48	4.28	.42
15	2	66	57	61	3.8	2	57	63	54	4.2	.4
16	2	72	60	65	3.74	2	62	66	60	4.12	.38
17	2	78	63	69	3.7	2	67	69	66	4.06	.36
18	2	84	66	73	3.67	2	72	72	72	4.	.33
19	1	86	71	78	3.73	2	77	75	78	4.1	.37
20	3	86	77	81	3.85	2	82	78	84	4.2	.35
21	3	86	83	84	3.93	2	87	81	90	4.28	.35
22	3	86	89	87	3.91	1	89	87	90	4.08	.17
23	3	86	95	90	3.74	1	91	93	90	4.03	.29
24	2	92	98	94	3.8	1	93	99	90	4.12	.32
25	2	98	101	98	3.91	1	95	105	90	4.2	.29
26	2	104	104	102	3.92	3	100	109	93	4.19	.27
27	2	110	107	106	3.96	3	105	113	96	4.18	.22
28	2	116	110	110	3.93	2	110	116	102	4.15	.22
29	2	122	113	114	3.89	2	115	119	108	4.11	.22
30	2	128	116	118	3.86	2	120	122	114	4.06	.20

[illegible]

	$\boxed{1}$	$\boxed{2}$	$\boxed{3}$
1	4	3	6
2	3	6	0
3	5	5	2

		ROW SUM					COL. SUM				
N	i	T_1	T_2	T_3	$\underline{v}$	j	T_1	T_2	T_3	$\bar{v}$	$\bar{v} - \underline{v}$
1	1	4	3	6	3.	2	3	6	5	6.	3.
2	2	7	9	6	3.	3	9	6	7	4.5	1.5
3	1	11	12	12	3.67	1	13	9	12	4.33	.66
4	1	15	15	18	3.75	1	17	12	17	4.25	.5
5	1	19	18	24	3.6	2	20	18	22	4.4	.8
6	3	24	23	26	3.82	2	23	24	27	4.5	.68
7	3	29	28	28	4.	2	26	30	32	4.55	.55
8	3	34	33	30	3.75	3	32	30	34	4.25	.5
9	3	39	38	32	3.55	3	38	30	36	4.22	.67
10	1	43	41	38	3.8	3	44	30	38	4.4	.6
11	1	47	44	44	4.	2	47	36	43	4.26	.26
12	1	51	47	50	3.92	2	50	42	48	4.17	.25
13	1	55	50	56	3.84	2	53	48	53	4.07	.23
14	1	59	53	62	3.79	2	56	54	58	4.14	.35
15	3	64	58	64	3.87	2	59	60	63	4.2	.33
16	3	69	63	66	3.93	2	62	66	68	4.25	.33
17	3	74	68	68	4.	2	65	72	73	4.29	.29
18	3	79	73	70	3.89	3	71	72	75	4.17	.28
19	3	84	78	72	3.79	3	77	72	77	4.05	.26
20	1	88	81	78	3.9	3	83	72	79	4.15	.25
21	1	92	84	84	4.	2	86	78	84	4.09	.09
22	1	96	87	90	3.95	2	89	84	89	4.04	.09
23	1	100	90	96	3.91	2	92	90	94	4.04	.13
24	3	105	95	98	3.96	2	95	96	99	4.12	.16
25	3	104	100	100	4.	2	98	102	104	4.16	.16
26	3	109	105	102	3.92	3	104	102	106	4.08	.16
27	3	114	110	104	3.85	3	110	102	108	4.07	.22
28	1	118	113	110	3.93	3	116	102	110	4.14	.21
29	1	122	116	116	4.	2	119	108	115	4.11	.11
30	1	126	119	122	3.97	2	122	114	120	4.07	.10

	1	2	3
1	4	6	3
2	5	2	5
3	3	0	6

		ROW SUM					COL. SUM				
N	i	T_1	T_2	T_3	$\underline{v}$	j	T_1	T_2	T_3	$\bar{v}$	$\bar{v} - \underline{v}$
1	1	4	6	3	3.	3	3	5	6	6.	3.
2	3	7	6	9	3.	2	9	7	6	4.5	1.5
3	1	11	12	12	3.67	1	13	12	9	4.33	.66
4	1	15	18	15	3.75	1	17	17	12	4.25	.5
5	1	19	24	18	3.6	3	20	22	18	4.4	.8
6	2	24	26	23	3.83	3	23	27	24	4.5	.67
7	2	29	28	28	4.	2	29	29	24	4.14	.14
8	1	33	34	31	3.88	3	32	34	30	4.25	.37
9	2	38	36	36	4.	2	38	36	30	4.12	.12
10	1	42	42	39	3.9	3	41	41	36	4.1	.2
11	1	46	48	42	3.81	3	44	46	42	4.18	.37
12	2	51	50	47	3.87	3	47	51	48	4.25	.38
13	2	56	52	52	4.	2	53	53	48	4.07	.07
14	1	60	58	55	3.93	3	56	58	54	4.14	.21
15	2	65	60	60	4.	2	62	60	54	4.13	.13
16	1	69	66	63	3.94	3	65	65	60	4.06	.12
17	1	73	72	66	3.88	3	68	70	66	4.12	.24
18	3	76	72	72	4.	2	74	72	66	4.11	.11
19	1	80	78	75	3.94	3	77	77	72	4.10	.16
20	1	84	84	78	3.9	3	80	82	78	4.1	.2
21	2	89	86	83	3.95	3	83	87	84	4.13	.18
22	2	94	88	88	4.	2	89	89	84	4.04	.04
23	1	98	94	91	3.95	3	92	94	90	4.08	.13
24	2	103	96	96	4.	2	98	96	90	4.08	.08
25	1	107	102	99	3.95	3	101	101	96	4.04	.09
26	1	111	108	102	3.92	3	104	106	102	4.08	.16
27	2	116	110	107	3.97	3	107	111	108	4.11	.14
28	2	121	112	112	4.	2	113	113	108	4.03	.03
29	1	125	118	115	3.97	3	116	118	114	4.07	.10
30	2	130	120	120	4.	2	122	120	114	4.07	.07

	1	$\boxed{2}$	$\boxed{3}$
1	6	0	3
2	3	6	4
3	5	2	5

	ROW SUM					COL. SUM					
N	i	T_1	T_2	T_3	$\underline{v}$	j	T_1	T_2	T_3	$\bar{v}$	$\bar{v} - \underline{v}$
1	1	6	0	3	0	2	0	6	2	6.	6.
2	2	9	6	7	3.	2	0	12	4	6.	3.
3	2	12	12	11	3.67	3	3	16	9	5.33	1.66
4	2	15	18	15	3.75	1	9	19	14	4.75	1.
5	2	18	24	19	3.6	1	15	22	19	4.4	.8
6	2	21	30	23	3.5	1	21	25	24	4.5	1.
7	2	24	36	27	3.57	1	27	28	29	4.14	.57
8	3	29	38	32	3.63	1	33	31	34	4.25	.62
9	3	34	40	37	3.78	1	39	34	39	4.33	.55
10	1	40	40	40	4.	1	45	37	44	4.5	.5
11	1	46	40	43	3.64	2	45	43	46	4.18	.54
12	3	51	42	48	3.5	2	45	49	48	4.08	.58
13	2	54	48	52	3.69	2	45	55	50	4.23	.54
14	2	57	54	56	3.85	2	45	61	52	4.35	.50
15	2	60	60	60	4.	1	51	64	57	4.27	.27
16	2	63	66	64	3.94	1	57	67	62	4.18	.24
17	2	66	72	68	3.88	1	63	70	67	4.12	.24
18	2	69	78	72	3.83	1	69	73	72	4.05	.22
19	2	72	84	76	3.79	1	75	76	77	4.05	.26
20	3	77	86	81	3.85	1	81	79	82	4.1	.25
21	3	82	88	86	3.9	1	87	82	87	4.14	.24
22	1	88	88	89	4.	1	93	85	92	4.22	.22
23	1	94	88	92	3.82	2	93	91	94	4.08	.26
24	3	99	90	97	3.75	2	93	97	96	4.03	.28
25	2	102	96	101	3.84	2	93	103	98	4.12	.26
26	2	105	102	105	3.92	2	93	109	100	4.19	.27
27	2	108	108	109	4.	1	99	112	105	4.14	.14
28	2	111	114	113	3.96	1	105	115	110	4.11	.15
29	2	114	120	117	3.93	1	111	118	115	4.07	.14
30	2	117	126	121	3.90	1	117	121	120	4.03	.13

	1	2	3
1	0	6	3
2	6	3	4
3	2	5	5

		ROW SUM					COL. SUM				
N	i	T_1	T_2	T_3	$\underline{v}$	j	T_1	T_2	T_3	$\bar{v}$	$\bar{v} - \underline{v}$
1	1	0	6	3	0	1	0	6	2	6.	6.
2	2	6	9	7	3.	1	0	12	4	6.	3.
3	2	12	12	11	3.67	3	3	16	9	5.33	1.66
4	2	18	15	15	3.75	2	9	19	14	4.75	1.
5	2	24	18	19	3.6	2	15	22	19	4.4	.8
6	2	30	21	23	3.5	2	21	25	24	4.17	.67
7	2	36	24	27	3.43	2	27	28	29	4.14	.71
8	3	38	29	32	3.62	2	33	31	34	4.25	.63
9	3	40	34	37	3.78	2	39	34	39	4.33	.55
10	1	40	40	40	4.	1	39	40	41	4.1	.1
11	3	42	45	45	3.82	1	39	46	43	4.17	.35
12	2	48	48	49	4.	1	39	52	45	4.33	.33
13	2	54	51	53	3.92	2	45	55	50	4.23	.31
14	2	60	54	57	3.85	2	51	58	55	4.13	.28
15	2	66	57	61	3.79	2	57	61	60	4.06	.27
16	2	72	60	65	3.75	2	63	64	65	4.06	.31
17	3	74	65	70	3.82	2	69	67	70	4.12	.30
18	3	76	70	75	3.89	2	75	70	75	4.17	.28
19	1	76	76	78	4.	1	75	76	77	4.05	.05
20	3	78	81	83	3.9	1	75	82	79	4.1	.2
21	2	84	84	87	4.	1	75	88	81	4.18	.18
22	2	90	87	91	3.95	2	81	91	86	4.13	.18
23	2	96	90	95	3.91	2	87	94	91	4.08	.17
24	2	102	93	99	3.87	2	93	97	96	4.03	.16
25	2	108	96	103	3.84	2	99	100	101	4.03	.19
26	3	110	101	108	3.88	1	99	106	103	4.07	.19
27	2	116	104	112	3.85	2	105	109	108	4.03	.18
28	2	122	107	116	3.82	2	111	112	113	4.03	.21
29	3	124	112	121	3.86	2	117	115	118	4.07	.21
30	3	126	117	126	3.9	2	123	118	123	4.1	.2

	1	2	3
1	6	3	4
2	0	6	3
3	2	5	5

		ROW SUM					COL. SUM				
N	i	T_1	T_2	T_3	$\underline{v}$	j	T_1	T_2	T_3	$\bar{v}$	$\bar{v} - \underline{v}$
1	1	6	3	4	3.	2	3	6	5	6.	3.
2	2	6	9	7	3.	1	9	6	7	4.5	1.5
3	1	12	12	11	3.67	3	13	9	12	4.33	.66
4	1	18	15	15	3.75	2	16	15	17	4.25	.5
5	3	20	20	20	4.	1	22	15	19	4.4	.4
6	1	26	23	24	3.83	2	25	21	24	4.17	.34
7	1	32	26	28	3.71	2	28	27	29	4.13	.42
8	3	34	31	33	3.87	2	31	33	34	4.25	.38
9	3	36	36	38	4.	1	37	33	36	4.11	.11
10	1	42	39	42	3.9	2	40	39	41	4.1	.2
11	3	44	44	47	4.	1	46	39	43	4.18	.18
12	1	50	47	51	3.92	2	49	45	48	4.08	.16
13	1	56	50	55	3.85	2	52	51	53	4.07	.22
14	3	58	55	60	3.93	2	55	57	58	4.13	.20
15	3	60	60	65	4.	1	61	57	60	4.06	.06
16	1	66	63	69	3.93	2	64	63	65	4.08	.15
17	3	68	68	74	4.	1	70	63	67	4.12	.12
18	1	74	71	78	3.95	2	73	69	72	4.05	.10
19	1	80	74	82	3.89	2	76	75	77	4.05	.16
20	3	82	79	87	3.95	2	79	81	82	4.1	.15
21	3	84	84	92	4.	1	85	81	84	4.05	.05
22	1	90	87	96	3.96	2	88	87	89	4.04	.08
23	3	92	92	101	4.	1	94	87	91	4.08	.08
24	1	98	95	105	3.95	2	97	93	96	4.03	.08
25	1	104	98	109	3.92	2	100	99	101	4.03	.11
26	3	106	103	114	3.96	2	103	105	106	4.07	.11
27	3	108	108	119	4.	1	109	105	108	4.03	.03
28	1	114	111	123	3.96	2	112	111	113	4.03	.07
29	3	116	116	128	4.	1	118	111	115	4.06	.06
30	1	122	119	132	3.97	2	121	117	120	4.03	.06

	1	2	3
1	0	6	3
2	2	5	5
3	6	3	4

	ROW SUM					COL. SUM					
N	i	T_1	T_2	T_3	$\underline{v}$	j	T_1	T_2	T_3	$\overline{v}$	$\overline{v} - \underline{v}$
1	1	0	6	3	0	1	0	2	6	6.	6.
2	3	6	9	7	3.	1	0	4	12	6.	3.
3	3	12	12	11	4.	3	3	9	16	5.33	1.33
4	3	18	15	15	3.75	2	9	14	19	4.75	1.
5	3	24	18	19	3.6	2	15	19	22	4.4	.8
6	3	30	21	23	3.5	2	21	24	25	4.18	.68
7	3	36	24	27	3.43	2	27	29	28	4.14	.71
8	2	38	29	32	3.63	2	33	34	31	4.25	.62
9	2	40	34	37	3.78	2	39	39	34	4.33	.55
10	1	40	40	40	4.	1	39	41	40	4.1	.1
11	2	42	45	45	3.81	1	39	43	46	4.18	.37
12	3	48	48	49	4.	1	39	45	52	4.33	.33
13	3	54	51	53	3.92	2	45	50	55	4.23	.31
14	3	60	54	57	3.86	2	51	55	58	4.14	.28
15	3	66	57	61	3.8	2	57	60	61	4.07	.27
16	3	72	60	65	3.75	2	63	65	64	4.06	.31
17	2	74	65	70	3.82	2	69	70	67	4.11	.39
18	2	76	70	75	3.89	2	75	75	70	4.17	.28
19	1	76	76	78	4.	1	75	77	76	4.05	.05
20	2	78	81	83	3.9	1	75	79	82	4.1	.2
21	3	84	84	87	4.	1	75	81	88	4.19	.19
22	1	84	90	90	3.82	1	75	83	94	4.27	.45
23	3	90	93	94	3.91	1	75	85	100	4.35	.44
24	3	96	96	98	4.	1	75	87	106	4.42	.42
25	3	102	99	102	3.96	2	81	92	109	4.36	.40
26	3	108	102	106	3.93	2	87	97	112	4.31	.38
27	3	114	105	110	3.89	2	93	102	115	4.26	.37
28	3	120	108	114	3.86	2	99	107	118	4.22	.36
29	3	126	111	118	3.83	2	105	112	121	4.17	.34
30	3	132	114	122	3.8	2	111	117	124	4.13	.33

[illegible]

	$\boxed{1}$	$\boxed{2}$	$\boxed{3}$
1	6	3	4
2	2	5	5
3	0	6	3

		ROW SUM					COL. SUM				
N	i	T1	T2	T3	$\underline{v}$	j	T1	T2	T3	$\bar{v}$	$\bar{v} - \underline{v}$
1	1	6	3	4	3.	2	3	5	6	6.	3.
2	3	6	9	7	3.	1	9	7	6	4.5	1.5
3	1	12	12	11	3.67	3	13	12	9	4.33	.66
4	1	18	15	15	3.75	2	16	17	15	4.25	.5
5	2	20	20	20	4.	1	22	19	15	4.4	.4
6	1	26	23	24	3.83	2	25	24	21	4.18	.35
7	1	32	26	28	3.71	2	28	29	27	4.14	.43
8	2	34	31	33	3.88	2	31	34	33	4.25	.37
9	2	36	36	38	4.	1	37	36	33	4.11	.11
10	1	42	39	42	3.9	2	40	41	39	4.1	.2
11	2	44	44	47	4.	1	46	43	39	4.18	.18
12	1	50	47	51	3.91	2	49	48	45	4.08	.17
13	1	56	50	55	3.84	2	52	53	51	4.07	.23
14	2	58	55	60	3.93	2	55	58	57	4.14	.21
15	2	60	60	65	4.	1	61	60	57	4.06	.06
16	1	66	63	69	3.93	2	64	65	63	4.06	.13
17	2	68	68	74	4.	1	70	67	63	4.12	.12
18	1	74	71	78	3.94	2	73	72	69	4.06	.12
19	1	80	74	82	3.89	2	76	77	75	4.05	.16
20	2	82	79	87	3.95	2	79	82	81	4.1	.15
21	2	84	84	92	4.	1	85	84	81	4.04	.04
22	1	90	87	96	3.95	2	88	89	87	4.04	.09
23	2	92	92	101	4.	1	94	91	87	4.08	.08
24	1	98	95	105	3.96	2	97	96	93	4.03	.07
25	1	104	98	109	3.92	2	100	101	99	4.03	.11
26	2	106	103	114	3.96	2	103	106	105	4.08	.12
27	2	108	108	119	4.	1	109	108	105	4.03	.03
28	1	114	111	123	3.97	2	112	113	111	4.03	.06
29	2	116	116	128	4.	1	118	115	111	4.07	.07
30	1	122	119	132	3.97	2	121	120	117	4.03	.06

	1	2	3
1)	0	3	6
2)	6	4	3
3)	2	5	5

	ROW SUM					COL. SUM					
N	i	T_1	T_2	T_3	$\underline{v}$	j	T_1	T_2	T_3	$\bar{v}$	$\bar{v} - \underline{v}$
1	1	0	3	6	0	1	0	6	2	6.	6.
2	2	6	7	9	3.	1	0	12	4	6.	3.
3	2	12	11	12	3.67	2	3	16	9	5.33	1.66
4	2	18	15	15	3.75	2	6	20	14	5.	1.25
5	2	24	19	18	3.6	3	12	23	19	4.6	1.
6	2	30	23	21	3.5	3	18	26	24	4.33	.83
7	2	36	27	24	3.43	3	24	29	29	4.14	.71
8	2	42	31	27	3.38	3	30	32	34	4.25	.87
9	3	44	36	32	3.55	3	36	35	39	4.33	.78
10	3	46	41	37	3.7	3	42	38	44	4.4	.70
11	3	48	46	42	3.81	3	48	41	49	4.45	.64
12	3	50	51	47	3.91	3	54	44	54	4.5	.59
13	1	50	54	53	3.85	1	54	50	56	4.31	.46
14	3	52	59	58	3.71	1	54	56	58	4.14	.43
15	3	54	64	63	3.6	1	54	62	60	4.13	.53
16	2	60	68	66	3.75	1	54	68	62	4.24	.49
17	2	66	72	69	3.88	1	54	74	64	4.35	.47
18	2	72	76	72	4.	1	54	80	66	4.44	.44
19	2	78	80	75	3.95	3	60	83	71	4.36	.41
20	2	84	84	78	3.9	3	66	86	76	4.3	.4
21	2	90	88	81	3.85	3	72	89	81	4.23	.38
22	2	96	92	84	3.81	3	78	92	86	4.18	.37
23	2	102	96	87	3.78	3	84	95	91	4.13	.35
24	2	108	100	90	3.75	3	90	98	96	4.08	.33
25	2	114	104	93	3.72	3	96	101	101	4.04	.32
26	2	120	108	96	3.69	3	102	104	106	4.07	.38
27	3	122	113	101	3.74	3	108	107	111	4.11	.37
28	3	124	118	106	3.79	3	114	110	116	4.14	.35
29	3	126	123	111	3.83	3	120	113	121	4.17	.34
30	3	128	128	116	3.86	3	126	116	126	4.19	.33

	$\boxed{1}$	$\boxed{2}$	$\boxed{3}$
1	6	4	3
2	0	3	6
3	2	5	5

		ROW SUM					COL. SUM				
N	i	T_1	T_2	T_3	$\underline{v}$	j	T_1	T_2	T_3	$\overline{v}$	$\overline{v}$ - $\underline{v}$
1	1	6	4	3	3.	3	3	6	5	6.	3.
2	2	6	7	9	3.	1	9	6	7	4.5	1.5
3	1	12	11	12	3.67	2	13	9	12	4.33	.66
4	1	18	15	15	3.75	2	17	12	17	4.25	.5
5	1	24	19	18	3.6	3	20	18	22	4.4	1.
6	3	26	24	23	3.83	3	23	24	27	4.5	.67
7	3	28	29	28	4.	1	29	24	29	4.13	.13
8	1	34	33	31	3.87	3	32	30	34	4.25	.38
9	3	36	38	36	4.	1	38	30	36	4.22	.22
10	1	42	42	39	3.9	3	41	36	41	4.1	.2
11	1	48	46	42	3.82	3	44	42	46	4.18	.36
12	3	50	51	47	3.92	3	47	48	51	4.25	.33
13	3	52	56	52	4.	1	53	48	53	4.08	.08
14	1	58	60	55	3.93	3	56	54	58	4.13	.20
15	3	60	65	60	4.	1	62	54	60	4.13	.13
16	1	66	69	63	3.93	3	65	60	65	4.06	.13
17	1	72	73	66	3.88	3	68	66	70	4.11	.23
18	3	74	78	71	3.95	3	71	72	75	4.16	.21
19	3	76	83	76	4.	1	77	72	77	4.05	.05
20	1	82	87	79	3.95	3	80	78	82	4.1	.15
21	3	84	92	84	4.	1	86	78	84	4.08	.08
22	1	90	96	87	3.96	3	89	84	89	4.04	.08
23	1	96	100	90	3.91	3	92	90	94	4.08	.17
24	3	98	105	95	3.96	3	95	96	99	4.12	.16
25	3	100	110	100	4.	1	101	96	101	4.03	.03
26	1	106	114	103	3.96	3	104	102	106	4.07	.11
27	3	108	119	108	4.	1	110	102	108	4.07	.07
28	1	114	123	111	3.96	3	113	108	113	4.03	.07
29	1	120	127	114	3.93	3	116	114	118	4.07	.14
30	3	122	132	119	3.97	3	119	120	123	4.1	.13

[illegible]

	$\boxed{1}$	$\boxed{2}$	$\boxed{3}$
1	0	3	6
2	2	5	5
3	6	4	3

		ROW SUM					COL. SUM				
N	i	T_1	T_2	T_3	$\underline{v}$	j	T_1	T_2	T_3	$\overline{v}$	$\overline{v} - \underline{v}$
1	1	0	3	6	0	1	0	2	6	6.	6.
2	3	6	7	9	3.	1	0	4	12	6.	3.
3	3	12	11	12	3.67	2	3	9	16	5.33	1.66
4	3	18	15	15	3.75	2	6	14	20	5.	1.25
5	3	24	19	18	3.6	3	12	19	23	4.6	1.
6	3	30	23	21	3.5	3	18	24	26	4.33	.83
7	3	36	27	24	3.43	3	24	29	29	4.14	.71
8	2	38	32	29	3.63	2	27	34	33	4.25	.62
9	2	40	37	34	3.77	3	33	39	36	4.33	.56
10	2	42	42	39	3.9	3	39	44	39	4.4	.5
11	2	44	47	44	4.	1	39	46	45	4.18	.18
12	2	46	52	49	3.83	1	39	48	51	4.25	.42
13	3	52	56	52	4.	1	39	50	57	4.39	.39
14	3	58	60	55	3.93	3	45	55	60	4.28	.35
15	3	64	64	58	3.86	3	51	60	63	4.2	.34
16	3	70	68	61	3.8	3	57	65	66	4.12	.32
17	3	76	72	64	3.76	3	63	70	69	4.11	.35
18	2	78	77	69	3.83	3	69	75	72	4.17	.34
19	2	80	82	74	3.89	3	75	80	75	4.21	.32
20	2	82	87	79	3.94	3	81	85	78	4.24	.3
21	2	84	92	84	4.	1	81	87	84	4.14	.14
22	2	86	97	89	3.91	1	81	89	90	4.08	.17
23	3	92	101	92	4.	1	81	91	96	4.17	.17
24	3	98	105	95	3.96	3	87	96	99	4.12	.16
25	3	104	109	98	3.91	3	93	101	102	4.08	.17
26	3	110	113	101	3.88	3	99	106	105	4.08	.2
27	2	112	118	106	3.93	3	105	111	108	4.11	.18
28	2	114	123	111	3.96	3	111	116	111	4.14	.18
29	2	116	128	116	4.	1	111	118	117	4.07	.07
30	2	118	133	121	3.93	1	111	120	123	4.09	.14

	$\boxed{1}$	$\boxed{2}$	$\boxed{3}$
1	6	4	3
2	2	5	5
3	0	3	6

		ROW SUM					COL. SUM				
N	i	T_1	T_2	T_3	$\underline{v}$	j	T_1	T_2	T_3	$\bar{v}$	$\bar{v} - \underline{v}$
1	1	6	4	3	3.	3	3	5	6	6.	3.
2	3	6	7	9	3.	1	9	7	6	4.5	1.5
3	1	12	11	12	3.67	2	13	12	9	4.33	.66
4	1	18	15	15	3.75	2	17	17	12	4.25	.5
5	1	24	19	18	3.6	3	20	22	18	4.4	.8
6	2	26	24	23	3.82	3	23	27	24	4.5	.68
7	2	28	29	28	4.	1	29	29	24	4.14	.14
8	1	34	33	31	3.88	3	32	34	30	4.25	.37
9	2	36	38	36	4.	1	38	36	30	4.22	.22
10	1	42	42	39	3.9	3	41	41	36	4.1	.2
11	1	48	46	42	3.82	3	44	46	42	4.18	.36
12	2	50	51	47	3.92	3	47	51	48	4.25	.33
13	2	52	56	52	4.	1	53	53	48	4.07	.07
14	1	58	60	55	3.93	3	56	58	54	4.14	.21
15	2	60	65	60	4.	1	62	60	54	4.13	.13
16	1	66	69	63	3.94	3	65	65	60	4.06	.12
17	1	72	73	66	3.88	3	68	70	66	4.12	.24
18	2	74	78	71	3.94	3	71	75	72	4.17	.23
19	2	76	83	76	4.	1	77	77	72	4.05	.05
20	1	82	87	79	3.95	3	80	82	78	4.1	.15
21	2	84	92	84	4.	1	86	84	78	4.09	.09
22	1	90	96	87	3.95	3	89	89	84	4.04	.09
23	1	96	100	90	3.91	3	92	94	90	4.08	.17
24	2	98	105	95	3.96	3	95	99	96	4.12	.16
25	2	100	110	100	4.	1	101	101	96	4.03	.03
26	1	106	114	103	3.97	3	104	106	102	4.07	.10
27	2	108	119	108	4.	1	110	108	102	4.07	.07
28	1	114	123	111	3.96	3	113	113	108	4.03	.07
29	1	120	127	114	3.93	3	116	118	114	4.07	.14
30	2	122	132	119	3.97	3	119	123	120	4.1	.13

	1	2	3
1	3	6	0
2	4	3	6
3	5	5	2

		ROW SUM					COL. SUM				
N	i	T_1	T_2	T_3	$\underline{v}$	j	T_1	T_2	T_3	$\bar{v}$	$\bar{v} - \underline{v}$
1	1	3	6	0	0	3	0	6	2	6.	6.
2	2	7	9	6	3.	3	0	12	4	6.	3.
3	2	11	12	12	3.67	1	3	16	9	5.33	1.66
4	2	15	15	18	3.75	1	6	20	14	5.	1.25
5	2	19	18	24	3.6	2	12	23	19	4.6	1.
6	2	23	21	30	3.5	2	18	26	24	4.33	.88
7	2	27	24	36	3.43	2	24	29	29	4.14	.71
8	2	31	27	42	3.87	2	30	32	34	4.5	.63
9	3	36	32	44	3.55	2	36	36	39	4.33	.78
10	3	41	37	46	3.7	2	42	39	44	4.4	.7
11	3	46	42	48	3.81	2	48	42	49	4.45	.64
12	3	51	47	50	3.92	2	54	45	54	4.5	.58
13	1	54	53	50	3.84	3	54	51	56	4.31	.47
14	3	59	58	52	3.71	3	54	57	58	4.14	.43
15	3	64	63	54	3.6	3	54	63	60	4.2	.6
16	2	68	66	60	3.75	3	54	69	62	4.32	.57
17	2	72	69	66	3.88	3	54	75	64	4.42	.54
18	2	76	72	72	4.	2	60	78	69	4.33	.33
19	2	80	75	78	3.95	2	66	81	74	4.26	.31
20	2	84	78	84	3.9	2	72	84	79	4.2	.3
21	2	88	81	90	3.85	2	78	87	84	4.13	.28
22	2	92	84	96	3.81	2	84	90	89	4.08	.27
23	2	96	87	102	3.78	2	90	93	94	4.08	.30
24	3	101	92	104	3.83	2	96	96	99	4.13	.30
25	3	106	97	106	3.88	2	102	99	104	4.17	.29
26	3	111	102	108	3.92	2	108	102	109	4.19	.27
27	3	116	107	110	3.96	2	114	105	114	4.22	.26
28	1	119	113	110	3.93	3	114	111	116	4.14	.21
29	3	124	118	112	3.86	3	114	117	118	4.07	.21
30	3	129	123	114	3.8	3	114	123	120	4.1	.3

	1	2	3
1	3	6	0
2	5	5	2
3	4	3	6

	ROW SUM					COL. SUM					
N	i	T_1	T_2	T_3	$\underline{v}$	j	T_1	T_2	T_3	$\bar{v}$	$\bar{v} - \underline{v}$
1	1	3	6	0	0	3	0	2	6	6.	6.
2	3	7	9	6	3.	3	0	4	12	6.	3.
3	3	11	12	12	3.67	1	3	9	16	5.33	1.66
4	3	15	15	18	3.75	1	6	14	20	5.	1.25
5	3	19	18	24	3.6	2	12	19	23	4.6	1.
6	3	23	21	30	3.5	2	18	24	26	4.33	.83
7	3	27	24	36	3.43	2	24	29	29	4.14	.71
8	2	32	29	38	3.63	2	30	34	32	4.25	.62
9	2	37	34	40	3.78	2	36	39	35	4.33	.55
10	2	42	39	42	3.9	2	42	44	38	4.4	.5
11	2	47	44	44	4.	2	48	49	41	4.45	.45
12	2	52	49	46	3.83	3	48	51	47	4.25	.47
13	2	57	54	48	3.69	3	48	53	53	4.07	.38
14	2	62	59	50	3.57	3	48	55	59	4.21	.64
15	3	66	62	56	3.73	3	48	57	65	4.33	.60
16	3	70	65	62	3.87	3	48	59	71	4.43	.56
17	3	74	68	68	4.	2	54	64	74	4.35	.35
18	3	78	71	74	3.95	2	60	69	77	4.27	.32
19	3	82	74	80	3.89	2	66	74	80	4.21	.32
20	3	86	77	86	3.85	2	72	79	83	4.15	.30
21	3	90	80	92	3.81	2	78	84	86	4.1	.29
22	3	94	83	98	3.77	2	84	89	89	4.04	.27
23	2	99	88	100	3.83	2	90	94	92	4.08	.25
24	2	104	93	102	3.87	2	96	99	95	4.12	.25
25	2	109	98	104	3.92	2	102	104	98	4.17	.25
26	2	114	103	106	3.96	2	108	109	101	4.19	.23
27	2	119	108	108	4.	2	114	114	104	4.22	.22
28	1	122	114	108	3.87	3	114	116	110	4.14	.27
29	2	127	119	110	3.79	3	114	118	116	4.07	.28
30	2	132	124	112	3.73	3	114	120	122	4.06	.33

	1	2	3
1	4	3	6
2	5	5	2
3	3	6	0

		ROW SUM				COL. SUM					
N	i	T_1	T_2	T_3	$\underline{v}$	j	T_1	T_2	T_3	$\overline{v}$	$\overline{v} - \underline{v}$
1	1	4	3	6	3.	2	3	5	6	6.	3.
2	3	7	9	6	3.	3	9	7	6	4.5	1.5
3	1	11	12	12	3.67	1	13	12	9	4.33	.66
4	1	15	15	18	3.75	1	17	17	12	4.25	.5
5	1	19	18	24	3.6	2	20	22	18	4.4	.8
6	2	24	23	26	3.83	2	23	27	24	4.5	.67
7	2	29	28	28	4.	2	26	32	30	4.57	.43
8	2	34	33	30	3.75	3	32	34	30	4.25	.5
9	2	39	38	32	3.33	3	38	36	30	4.22	.89
10	1	43	41	38	3.8	3	44	38	30	4.4	.6
11	1	47	44	44	4.	2	47	43	36	4.26	.26
12	1	51	47	50	3.92	2	50	48	42	4.18	.26
13	1	55	50	56	3.85	2	53	53	48	4.08	.23
14	1	59	53	62	3.78	2	56	58	54	4.14	.36
15	2	64	58	64	3.87	2	59	63	60	4.2	.33
16	3	69	63	66	3.94	2	62	68	66	4.24	.3
17	2	74	68	68	4.	2	65	73	72	4.28	.28
18	2	79	73	70	3.89	3	71	75	72	4.17	.28
19	2	84	78	72	3.78	3	77	77	72	4.06	.28
20	1	88	81	78	3.9	3	83	79	72	4.15	.25
21	1	92	84	84	4.	2	86	84	78	4.09	.09
22	1	96	87	90	3.95	2	89	89	84	4.04	.09
23	1	100	90	96	3.91	2	92	94	90	4.08	.17
24	2	105	95	98	3.95	2	95	99	96	4.12	.17
25	2	110	100	100	4.	2	98	104	102	4.17	.17
26	2	115	105	102	3.92	3	104	106	102	4.07	.15
27	2	120	110	104	3.85	3	110	108	102	4.07	.22
28	1	124	113	110	3.93	3	116	110	102	4.14	.21
29	1	128	116	116	4.	2	119	115	108	4.1	.1
30	1	132	119	122	3.96	2	122	120	114	4.07	.11

	$\boxed{1}$	$\boxed{2}$	$\boxed{3}$
1	3	0	6
2	5	2	5
3	4	6	3

	ROW SUM					COL. SUM					
N	i	T_1	T_2	T_3	$\underline{v}$	j	T_1	T_2	T_3	$\bar{v}$	$\bar{v} - \underline{v}$
1	1	3	0	6	0	2	0	2	6	6.	6.
2	3	7	6	9	3.	2	0	4	12	6.	3.
3	3	11	12	12	3.67	1	3	9	16	5.33	2.66
4	3	15	18	15	3.75	1	6	14	20	5.	1.25
5	3	19	24	18	3.6	3	12	19	23	4.6	1.
6	3	23	30	21	3.5	3	18	24	26	4.33	.83
7	3	27	36	24	3.43	3	24	29	29	4.14	.71
8	2	32	38	29	3.63	3	30	34	32	4.25	.62
9	2	37	40	34	3.78	3	36	39	35	4.33	.55
10	2	42	42	39	3.9	3	42	44	38	4.4	.5
11	2	47	44	44	4.	2	42	46	44	4.18	.18
12	2	52	46	49	3.84	2	42	48	50	4.16	.32
13	3	56	52	52	4.	2	42	50	56	4.31	.31
14	3	60	58	55	3.93	3	48	55	59	4.21	.28
15	3	64	64	58	3.87	3	54	60	62	4.12	.25
16	3	68	70	61	3.81	3	60	65	65	4.06	.25
17	2	73	72	66	3.88	3	66	70	68	4.12	.24
18	2	78	74	71	3.94	3	72	75	71	4.17	.23
19	2	83	76	76	4.	2	72	77	77	4.06	.06
20	2	88	78	81	3.9	2	72	79	83	4.15	.25
21	3	92	84	84	4.	2	72	81	89	4.23	.23
22	3	96	90	87	3.96	3	78	86	92	4.18	.22
23	3	100	96	90	3.91	3	84	91	95	4.13	.22
24	3	104	102	93	3.87	3	90	96	98	4.08	.21
25	3	108	108	96	3.84	3	96	101	101	4.04	.20
26	2	113	110	101	3.89	3	102	106	104	4.08	.19
27	2	118	112	106	3.92	3	108	111	107	4.12	.20
28	2	123	114	111	3.97	2	108	113	113	4.03	.06
29	2	128	116	116	4.	2	108	115	119	4.1	.1
30	3	132	122	119	3.96	3	114	120	122	4.07	.11

	$\boxed{1}$	$\boxed{2}$	$\boxed{3}$
1	3	0	6
2	4	6	3
3	5	2	5

		ROW SUM					COL. SUM				
N	i	T_1	T_2	T_3	$\underline{v}$	j	T_1	T_2	T_3	$\bar{v}$	$\bar{v} - \underline{v}$
1	1	3	0	6	0	2	0	6	2	6.	6.
2	2	7	6	9	3.	2	0	12	4	6.	3.
3	2	11	12	12	3.67	1	3	16	9	5.33	1.66
4	2	15	18	15	3.75	1	6	20	14	5.	1.25
5	2	19	24	18	3.6	3	12	23	19	4.6	1.
6	2	23	30	21	3.5	3	18	26	24	4.33	.83
7	2	27	36	24	3.43	3	24	29	29	4.14	.71
8	2	31	42	27	3.37	3	30	32	34	4.25	.88
9	3	36	44	32	3.56	3	36	35	39	4.33	.77
10	3	41	46	37	3.7	3	42	38	44	4.4	.7
11	3	46	48	42	3.82	3	48	41	49	4.45	.63
12	3	51	50	47	3.92	3	54	44	54	4.5	.58
13	1	54	50	53	3.84	2	54	50	56	4.31	.47
14	3	59	52	58	3.71	2	54	56	58	4.14	.43
15	2	63	58	61	3.87	2	54	62	60	4.13	.26
16	2	67	64	64	4.	2	54	68	62	4.25	.25
17	2	71	70	67	3.94	3	60	71	67	4.17	.23
18	2	75	76	70	3.89	3	66	74	72	4.11	.22
19	2	79	82	73	3.84	3	72	77	77	4.05	.21
20	2	83	88	76	3.8	3	78	80	82	4.1	.2
21	3	88	90	81	3.85	3	84	83	87	4.13	.28
22	3	93	92	86	3.91	3	90	86	92	4.18	.27
23	3	98	94	91	3.95	3	96	89	97	4.22	.27
24	3	103	96	96	4.	2	96	95	99	4.12	.12
25	3	108	98	101	3.91	2	96	101	101	4.03	.12
26	2	112	104	104	4.	2	96	107	103	4.12	.12
27	2	116	110	107	3.96	3	102	110	108	4.07	.11
28	2	120	116	110	3.93	3	108	113	113	4.03	.10
29	2	124	122	113	3.90	3	114	116	118	4.07	.17
30	3	129	124	118	3.93	3	120	119	123	4.10	.17

	$\boxed{1}$	$\boxed{2}$	$\boxed{3}$
(1)	4	6	3
(2)	3	0	6
(3)	5	2	5

	ROW SUM					COL. SUM					
N	i	T_1	T_2	T_3	$\underline{v}$	j	T_1	T_2	T_3	$\bar{v}$	$\bar{v} - \underline{v}$
1	1	4	6	3	3.	3	3	6	5	6.	3.
2	2	7	6	9	3.	2	9	6	7	4.5	1.5
3	1	11	12	12	3.66	1	13	9	12	4.33	.66
4	1	15	18	15	3.75	1	17	12	17	4.25	.5
5	1	19	24	18	3.6	3	20	18	22	4.4	.8
6	3	24	26	23	3.83	3	23	24	27	4.5	.67
7	3	29	28	28	4.	2	29	24	29	4.14	.14
8	1	33	34	31	3.88	3	32	30	34	4.25	.37
9	3	38	36	36	4.	2	38	30	36	4.22	.22
10	1	42	42	39	3.9	3	41	36	41	4.1	.2
11	1	46	48	42	3.82	3	44	42	46	4.18	.36
12	3	51	50	47	3.92	3	47	48	51	4.25	.33
13	3	56	52	52	4.	2	53	48	53	4.07	.07
14	1	60	58	55	3.93	3	56	54	58	4.14	.21
15	3	65	60	60	4.	2	62	54	60	4.13	.13
16	1	69	66	63	3.94	3	65	60	65	4.06	.12
17	1	73	72	66	3.88	3	68	66	70	4.12	.24
18	3	78	74	71	3.94	3	71	72	75	4.17	.23
19	3	83	76	76	4.	2	77	72	77	4.06	.06
20	1	87	82	79	3.95	3	80	78	82	4.1	.15
21	3	92	84	84	4.	2	86	78	84	4.09	.09
22	1	96	90	87	3.96	3	89	84	89	4.03	.07
23	1	100	96	90	3.91	3	92	90	94	4.08	.17
24	3	105	98	95	3.96	3	95	96	99	4.12	.16
25	3	110	100	100	4.	2	101	96	101	4.04	.04
26	1	114	106	103	3.96	3	104	102	106	4.07	.11
27	3	119	108	108	4.	2	110	102	108	4.07	.07
28	1	123	114	111	3.97	3	113	108	113	4.03	.06
29	1	127	120	114	3.93	3	116	114	118	4.07	.14
30	3	132	122	119	3.97	3	119	120	123	4.1	.13

	1	2	3
1	5	2	5
2	4	6	3
3	3	0	6

N	i	ROW SUM T_1	T_2	T_3	$\underline{v}$	j	COL. SUM T_1	T_2	T_3	$\bar{v}$	$\bar{v} - \underline{v}$
1	1	5	2	5	2.	2	2	6	0	6.	4.
2	2	9	8	8	4.	2	4	12	0	6.	2.
3	2	13	14	11	3.67	3	9	15	6	5.	1.33
4	3	16	14	17	3.5	2	11	21	6	5.25	1.75
5	2	20	20	20	4.	1	16	25	9	5.	1.
6	2	24	26	23	3.82	3	21	28	15	4.67	.85
7	2	28	32	26	3.71	3	26	31	21	4.43	.72
8	2	32	38	29	3.62	3	31	34	27	4.25	.63
9	2	36	44	32	3.55	3	36	37	33	4.11	.56
10	2	40	50	35	3.5	3	41	40	39	4.1	.6
11	1	45	52	40	3.63	3	46	43	45	4.18	.55
12	1	50	54	45	3.75	3	51	46	51	4.25	.5
13	1	55	56	50	3.84	3	56	49	57	4.39	.55
14	3	58	56	56	4.	2	58	55	57	4.13	.13
15	1	63	58	61	3.84	2	60	61	57	4.06	.22
16	2	67	64	64	4.	2	62	67	57	4.18	.18
17	2	71	70	67	3.94	3	67	70	63	4.12	.18
18	2	75	76	70	3.89	3	72	73	69	4.06	.17
19	2	79	82	73	3.84	3	77	76	75	4.05	.21
20	1	84	84	78	3.9	3	82	79	81	4.1	.2
21	1	89	86	83	3.94	3	87	82	87	4.13	.19
22	1	94	88	88	4.	2	89	88	87	4.04	.04
23	1	99	90	93	3.91	2	91	94	87	4.08	.17
24	2	103	96	96	4.	2	93	100	87	4.17	.17
25	2	107	102	99	3.96	3	98	103	93	4.12	.16
26	2	111	108	102	3.93	3	103	106	99	4.08	.15
27	2	115	114	105	3.89	3	108	109	105	4.03	.14
28	2	119	120	108	3.85	3	113	112	111	4.03	.18
29	1	124	122	113	3.89	3	118	115	117	4.07	.18
30	1	129	124	118	3.93	3	123	118	123	4.1	.17

	1	2	3
1	6	3	0
2	3	4	6
3	5	5	2

	ROW SUM					COL. SUM					
N	i	T_1	T_2	T_3	$\underline{v}$	j	T_1	T_2	T_3	$\bar{v}$	$\bar{v} - \underline{v}$
1	1	6	3	0	0	3	0	6	2	6.	6.
2	2	9	7	6	3.	3	0	12	4	6.	3.
3	2	12	11	12	3.67	2	3	16	9	5.33	2.67
4	2	15	15	18	3.75	1	9	19	14	4.75	1.
5	2	18	19	24	3.6	1	15	22	19	4.4	.8
6	2	21	23	30	3.5	1	21	25	24	4.18	.68
7	2	24	27	36	3.43	1	27	28	29	4.14	.71
8	3	29	32	38	3.63	1	33	31	34	4.25	.62
9	3	34	37	40	3.77	1	39	34	39	4.33	.56
10	1	40	40	40	4.	1	45	37	44	4.5	.5
11	1	46	43	40	3.64	3	45	43	46	4.18	.54
12	3	51	48	42	3.5	3	45	49	48	4.08	.58
13	2	54	52	48	3.69	3	45	55	50	4.23	.54
14	2	57	56	54	3.85	3	45	61	52	4.36	.51
15	2	60	60	60	4.	1	51	64	57	4.27	.25
16	2	63	64	66	3.94	1	57	67	62	4.18	.24
17	2	66	68	72	3.88	1	63	70	67	4.12	.24
18	2	69	72	78	3.84	1	69	73	72	4.06	.20
19	2	72	76	84	3.78	1	75	76	77	4.05	.27
20	3	77	81	86	3.85	1	81	79	82	4.10	.25
21	3	82	86	88	3.90	1	87	82	87	4.14	.24
22	1	88	89	88	4.	1	93	85	92	4.22	.22
23	1	94	92	88	3.82	3	93	91	94	4.08	.20
24	3	99	97	90	3.75	3	93	97	96	4.03	.28
25	2	102	101	96	3.84	3	93	103	98	4.12	.28
26	2	105	105	102	3.93	3	93	109	100	4.18	.25
27	2	108	109	108	4.	1	99	112	105	4.14	.14
28	2	111	113	114	3.96	1	105	115	110	4.11	.15
29	2	114	117	120	3.93	1	111	118	115	4.07	.14
30	2	117	121	126	3.89	1	117	121	120	4.03	.14

	$\boxed{1}$	$\boxed{2}$	$\boxed{3}$
1	6	3	0
2	5	5	2
3	3	4	6

		ROW SUM					COL. SUM				
N	i	T_1	T_2	T_3	$\underline{v}$	j	T_1	T_2	T_3	$\bar{v}$	$\bar{v} - \underline{v}$
1	1	6	3	0	0	3	0	2	6	6.	6.
2	3	9	7	6	3.	3	0	4	12	6.	3.
3	3	12	11	12	3.67	2	3	9	16	5.33	2.67
4	3	15	15	18	3.75	1	9	14	19	4.75	1.
5	3	18	19	24	3.6	1	15	19	22	4.4	.8
6	3	21	23	30	3.5	1	21	24	25	4.17	.67
7	3	24	27	36	3.42	1	27	29	28	4.14	.72
8	2	29	32	38	3.63	1	33	34	31	4.25	.62
9	2	34	37	40	3.78	1	39	39	34	4.33	.55
10	1	40	40	40	4.	1	45	44	37	4.5	.5
11	1	46	43	40	3.64	3	45	46	43	4.18	.54
12	2	51	48	42	3.5	3	45	48	49	4.08	.58
13	3	54	52	48	3.69	3	45	50	55	4.23	.54
14	3	57	56	54	3.85	3	45	52	61	4.35	.5
15	3	60	60	60	4.	1	51	57	64	4.27	.27
16	3	63	64	66	3.94	1	57	62	67	4.18	.24
17	3	66	68	72	3.87	1	63	67	70	4.11	.24
18	3	69	72	78	3.83	1	69	72	73	4.06	.23
19	3	72	76	84	3.79	1	75	77	76	4.05	.26
20	2	77	81	86	3.85	1	81	82	79	4.10	.25
21	2	82	86	88	3.90	1	87	87	82	4.13	.23
22	1	88	89	88	4.	1	93	92	85	4.22	.22
23	1	94	92	88	3.82	3	93	94	91	4.08	.20
24	2	99	97	90	3.75	3	93	96	97	4.03	.28
25	3	102	101	96	3.84	3	93	98	103	4.13	.29
26	3	105	105	102	3.93	3	93	100	109	4.19	.26
27	3	108	109	108	4.	1	99	105	112	4.15	.15
28	3	111	113	114	3.96	1	105	110	115	4.11	.15
29	3	114	117	120	3.93	1	111	115	118	4.10	.17
30	3	117	121	126	3.90	1	117	120	121	4.03	.13

[illegible]

	1	2	3
1	3	4	6
2	5	5	2
3	6	3	0

		ROW SUM					COL. SUM				
N	i	T_1	T_2	T_3	$\underline{v}$	j	T_1	T_2	T_3	$\overline{v}$	$\overline{v} - \underline{v}$
1	1	3	4	6	3.	1	3	5	6	6.	3.
2	3	9	7	6	3.	3	9	7	6	4.5	1.5
3	1	12	11	12	3.67	2	13	12	9	4.33	.66
4	1	15	15	18	3.75	1	16	17	15	4.25	.5
5	2	20	20	20	4.	1	19	22	21	4.4	.4
6	2	25	25	22	3.67	3	25	24	21	4.17	.4
7	1	28	29	28	4.	1	28	29	27	4.14	.14
8	2	33	34	30	3.75	3	34	31	27	4.12	.37
9	1	36	38	36	4.	1	37	36	33	4.11	.11
10	1	39	42	42	3.9	1	40	41	39	4.1	.2
11	2	44	47	44	4.	1	43	46	45	4.18	.18
12	2	49	52	46	3.82	3	49	48	45	4.08	.2
13	1	52	56	52	4.	1	52	53	51	4.07	.07
14	2	57	61	54	3.85	3	58	55	51	4.14	.29
15	1	60	65	60	4.	1	61	60	57	4.06	.06
16	1	63	69	66	3.94	1	64	65	63	4.07	.13
17	2	68	74	68	4.	1	67	70	69	4.12	.12
18	2	73	79	70	3.88	3	73	72	69	4.05	.17
19	1	76	83	76	4.	1	76	77	75	4.05	.05
20	2	81	88	78	3.9	3	82	79	75	4.1	.2
21	1	84	92	84	4.	1	85	84	81	4.04	.04
22	1	87	96	90	3.95	1	88	89	87	4.03	.08
23	2	92	101	92	4.	1	91	94	93	4.08	.08
24	2	97	106	94	3.91	3	97	96	93	4.03	.12
25	1	100	110	100	4.	1	100	101	99	4.03	.03
26	2	105	115	102	3.92	3	106	103	99	4.07	.15
27	1	108	119	108	4.	1	109	108	105	4.03	.03
28	1	111	123	114	3.97	1	112	113	111	4.03	.06
29	2	116	128	116	4.	1	115	118	117	4.07	.07
30	2	121	133	118	3.93	3	121	120	117	4.03	.1

	$\boxed{1}$	$\boxed{2}$	$\boxed{3}$
1	3	4	6
2	6	3	0
3	5	5	2

		ROW SUM					COL. SUM				
N	i	T_1	T_2	T_3	$\underline{v}$	j	T_1	T_2	T_3	$\overline{v}$	$\overline{v} - \underline{v}$
1	1	3	4	6	3.	1	3	6	5	6.	3.
2	2	9	7	6	3.	3	9	6	7	4.5	1.5
3	1	12	11	12	3.67	2	13	9	12	4.33	.66
4	1	15	15	18	3.75	1	16	15	17	4.25	.5
5	3	20	20	20	4.	1	19	21	22	4.4	.4
6	3	25	25	22	4.67	3	25	21	24	4.17	.4
7	1	28	29	28	4.	1	28	27	29	4.14	.14
8	3	33	34	30	3.75	3	34	27	31	4.12	.37
9	1	36	38	36	4.	1	37	33	36	4.11	.11
10	1	39	42	42	3.9	1	40	39	41	4.1	.2
11	3	44	47	44	4.	1	43	45	46	4.18	.18
12	3	49	52	46	3.82	3	49	45	48	4.08	.2
13	1	52	56	52	4.	1	52	51	53	4.07	.07
14	3	57	61	54	3.85	3	58	51	55	4.14	.29
15	1	60	65	60	4.	1	61	57	60	4.06	.06
16	1	63	69	66	3.94	1	64	63	65	4.07	.13
17	3	68	74	68	4.	1	67	69	70	4.12	.12
18	3	73	79	70	3.88	3	73	69	72	4.05	.17
19	1	76	83	76	4.	1	76	75	77	4.05	.05
20	3	81	88	78	3.9	3	82	75	79	4.1	.2
21	1	84	92	84	4.	1	85	81	84	4.04	.04
22	1	87	96	90	3.95	1	88	87	89	4.03	.08
23	3	92	101	92	4.	1	91	93	94	4.08	.08
24	3	97	106	94	3.91	3	97	93	96	4.03	.12
25	1	100	110	100	4.	1	100	99	101	4.03	.03
26	3	105	115	102	3.92	3	106	99	103	4.07	.15
27	1	108	119	108	4.	1	109	105	108	4.03	.03
28	1	111	123	114	3.97	1	112	111	113	4.03	.06
29	3	116	128	116	4.	1	115	117	118	4.07	.07
30	3	121	133	118	3.93	3	121	117	120	4.03	.1

	1	2	3
1	3	6	4
2	5	2	5
3	6	0	3

		ROW SUM					COL. SUM				
N	i	T_1	T_2	T_3	$\underline{v}$	j	T_1	T_2	T_3	$\bar{v}$	$\bar{v} - \underline{v}$
1	1	3	6	4	3.	1	3	5	6	6.	3.
2	3	9	6	7	3.	2	9	7	6	4.5	1.5
3	1	12	12	11	3.67	3	13	12	9	4.33	.66
4	1	15	18	15	3.75	1	16	17	15	4.25	.5
5	2	20	20	20	4.	1	19	22	21	4.4	.4
6	2	25	22	25	3.67	2	25	24	21	4.17	.5
7	1	28	28	29	4.	1	28	29	27	4.14	.14
8	2	33	30	34	3.75	2	34	31	27	4.25	.5
9	1	36	36	38	4.	1	37	36	33	4.11	.11
10	1	39	42	42	3.9	1	40	41	39	4.1	.2
11	2	44	44	47	4.	1	43	46	45	4.18	.18
12	2	49	46	52	3.83	2	49	48	45	4.08	.25
13	1	52	52	56	4.	1	52	53	51	4.03	.03
14	2	57	54	61	3.85	2	58	55	51	4.14	.29
15	1	60	60	65	4.	1	61	60	57	4.06	.06
16	1	63	66	69	3.94	1	64	65	63	4.07	.13
17	2	68	68	74	4.	1	67	70	69	4.11	.11
18	2	73	70	79	3.89	2	73	72	69	4.06	.17
19	1	76	76	83	4.	1	76	77	75	4.05	.05
20	2	81	78	88	3.9	2	82	79	75	4.1	.2
21	1	84	84	92	4.	1	85	84	81	4.05	.05
22	1	87	90	96	3.95	1	88	89	87	4.04	.09
23	2	92	92	101	4.	1	91	94	93	4.08	.08
24	2	97	94	106	3.91	2	97	96	93	4.03	.12
25	1	100	100	110	4.	1	100	101	99	4.03	.03
26	2	105	102	115	3.92	2	106	103	99	4.07	.15
27	1	108	108	119	4.	1	109	108	105	4.03	.03
28	1	111	114	123	3.97	1	112	113	111	4.03	.06
29	2	116	116	128	4.	1	115	118	117	4.07	.07
30	2	121	118	133	3.93	2	121	120	117	4.03	.1

	1	2	3
1	3	6	4
2	6	0	3
3	5	2	5

		ROW SUM					COL. SUM				
N	i	T1	T2	T3	$\underline{v}$	j	T1	T2	T3	$\bar{v}$	$\bar{v} - \underline{v}$
1	1	3	6	4	3.	1	3	6	5	6.	3.
2	2	9	6	7	3.	2	9	6	7	4.5	1.5
3	1	12	12	11	3.67	3	13	9	12	4.33	.66
4	1	15	18	15	3.75	1	16	15	17	4.25	.5
5	3	20	20	20	4.	1	19	21	22	4.4	.4
6	3	25	22	25	3.67	2	25	21	24	4.17	.5
7	1	28	28	29	4.	1	28	27	29	4.14	.14
8	3	33	30	34	3.75	2	34	27	31	4.25	.5
9	1	36	36	38	4.	1	37	33	36	4.11	.11
10	1	39	42	42	3.9	1	40	39	41	4.1	.2
11	3	44	44	47	4.	1	43	45	46	4.18	.18
12	3	49	46	52	3.83	2	49	45	48	4.08	.25
13	1	52	52	56	4.	1	52	51	53	4.03	.03
14	3	57	54	61	3.85	2	58	51	55	4.14	.29
15	1	60	60	65	4.	1	61	57	60	4.06	.06
16	1	63	66	69	3.94	1	64	63	65	4.07	.13
17	3	68	68	74	4.	1	67	69	70	4.11	.11
18	3	73	70	79	3.89	2	73	69	72	4.06	.17
19	1	76	76	83	4.	1	76	75	77	4.05	.05
20	3	81	78	88	3.9	2	82	75	79	4.1	.2
21	1	84	84	92	4.	1	85	81	84	4.05	.05
22	1	87	90	96	3.95	1	88	87	89	4.04	.09
23	3	92	92	101	4.	1	91	93	94	4.08	.08
24	3	97	94	106	3.91	2	97	93	96	4.03	.12
25	1	100	100	110	4.	1	100	99	101	4.03	.03
26	3	105	102	115	3.92	2	106	99	103	4.07	.15
27	1	108	108	119	4.	1	109	105	108	4.03	.03
28	1	111	114	123	3.97	1	112	111	113	4.03	.06
29	3	116	116	128	4.	1	115	117	118	4.07	.07
30	3	121	118	133	3.93	2	121	117	120	4.03	.1

	1	2	3
1	5	4	3
2	5	3	6
3	2	6	0

	ROW SUM					COL. SUM					
N	i	T_1	T_2	T_3	$\underline{v}$	j	T_1	T_2	T_3	$\bar{v}$	$\bar{v} - \underline{v}$
1	1	5	4	3	3.	3	3	6	0	6.	3.
2	2	10	7	9	3.5	2	7	9	6	4.5	1.
3	2	15	10	15	3.33	2	11	12	12	4.	.67
4	2	20	13	21	3.25	2	15	15	18	4.5	1.25
5	3	22	19	21	3.8	2	19	18	24	4.8	1.
6	3	24	25	21	3.5	3	22	24	24	4.	.5
7	2	29	28	27	3.86	3	25	30	24	4.28	.42
8	2	34	31	33	3.87	2	29	33	30	4.12	.25
9	2	39	34	39	3.77	2	33	36	36	4.	.23
10	2	44	37	45	3.7	2	37	39	42	4.2	.5
11	3	46	43	45	3.91	2	41	42	48	4.37	.46
12	3	48	49	45	3.75	3	44	48	48	4.	.25
13	2	53	52	51	3.92	3	47	54	48	4.17	.25
14	2	58	53	57	3.79	2	51	57	54	4.07	.28
15	2	63	56	63	3.73	2	55	60	60	4.	.27
16	2	68	59	69	3.68	2	59	63	66	4.12	.44
17	3	70	65	69	3.82	2	63	66	72	4.23	.41
18	3	72	71	69	3.83	3	66	72	72	4.	.17
19	2	77	74	75	3.89	2	70	75	78	4.11	.22
20	3	79	80	75	3.75	3	73	81	78	4.05	.30
21	2	84	83	81	3.85	3	76	87	78	4.14	.29
22	2	89	86	87	3.91	2	80	90	84	4.08	.17
23	2	94	89	93	3.87	2	84	93	90	4.04	.17
24	2	99	92	96	3.83	2	88	96	96	4.	.17
25	2	104	95	102	3.8	2	92	99	102	4.08	.28
26	3	106	101	102	3.89	2	96	102	108	4.16	.27
27	3	108	107	102	3.78	3	99	108	108	4.	.22
28	2	113	110	108	3.86	3	102	114	108	4.07	.21
29	2	118	113	114	3.9	2	106	117	114	4.03	.13
30	2	123	116	120	3.86	2	110	120	120	4.	.14

	1	2	3
1	6	7	1
2	6	3	2
3	2	4	5

		ROW SUM					COL. SUM				
N	i	T_1	T_2	T_3	$\underline{v}$	j	T_1	T_2	T_3	$\bar{v}$	$\bar{v} - \underline{v}$
1	1	6	7	1	1.	3	1	2	5	5.	4.
2	3	8	11	6	3.	3	2	4	10	5.	2.
3	3	10	15	11	3.33	1	8	10	12	4.	.67
4	3	12	19	16	3.	1	14	16	14	4.	1.
5	2	18	22	18	3.6	1	20	22	16	4.4	.8
6	2	24	25	20	3.33	3	21	24	21	4.	.67
7	2	30	28	22	3.14	3	22	26	26	3.72	.58
8	2	36	31	24	3.	3	23	28	31	3.87	.87
9	3	38	35	29	3.22	3	24	30	36	4.	.78
10	3	40	39	34	3.4	3	25	32	41	4.1	.7
11	3	42	43	39	3.55	3	26	34	46	4.18	.63
12	3	44	47	44	3.68	1	32	40	48	4.	.32
13	3	46	51	49	3.57	1	38	46	50	3.85	.28
14	3	48	55	54	3.42	1	44	52	52	3.71	.29
15	2	54	58	56	3.6	1	45	54	57	3.79	.19
16	3	56	62	61	3.49	1	51	60	59	3.75	.26
17	2	56	65	63	3.29	1	57	66	61	3.88	.59
18	2	62	68	65	3.39	1	63	72	63	4.	.61
19	2	68	71	67	3.52	3	64	74	68	3.88	.36
20	2	74	74	69	3.45	3	65	76	73	3.8	.35
21	2	80	77	71	3.37	3	66	78	78	3.71	.34
22	2	86	80	73	3.32	3	68	82	83	3.76	.44
23	3	88	84	78	3.39	3	69	84	88	3.82	.43
24	3	90	88	83	3.46	3	70	86	93	3.87	.41
25	3	92	92	88	3.51	3	71	88	98	3.91	.40
26	3	94	96	93	3.58	3	73	92	103	3.96	.38
27	3	96	100	98	3.55	1	79	98	105	3.88	.33
28	3	98	104	103	3.50	1	85	104	107	3.82	.32
29	3	100	108	108	3.44	1	91	110	109	3.76	.32
30	2	106	111	110	3.53	1	97	116	111	3.87	.34

	1	2	3
1	2	6	0
2	5	3	6
3	5	4	3

		ROW SUM					COL. SUM				
N	i	T_1	T_2	T_3	$\underline{v}$	j	T_1	T_2	T_3	$\overline{v}$	$\overline{v} - \underline{v}$
1	1	2	6	0	0	3	0	6	3	6.	6.
2	2	7	9	6	3.	3	0	12	6	6.	3.
3	2	12	12	12	4.	1	2	17	11	5.67	1.67
4	2	17	15	18	3.75	2	8	20	15	5.	1.25
5	2	22	18	24	3.6	2	14	23	19	4.6	1.
6	2	27	21	30	3.5	2	20	26	23	4.33	.83
7	2	32	24	36	3.42	2	26	29	27	4.14	.72
8	2	37	27	42	3.37	2	32	32	31	4.	.63
9	1	39	33	42	3.68	2	38	35	35	4.22	.54
10	1	41	39	42	3.9	2	44	38	39	4.4	.5
11	1	43	45	42	3.81	3	44	44	42	4.	.19
12	1	45	51	42	3.5	3	44	50	45	4.17	.67
13	2	50	54	48	3.69	3	44	56	48	4.31	.62
14	2	55	57	54	3.85	3	44	62	51	4.43	.58
15	2	60	60	60	4.	1	46	67	56	4.46	.46
16	2	65	63	66	3.94	2	52	70	60	4.37	.43
17	2	70	66	72	3.88	2	58	73	64	4.29	.41
18	2	75	69	78	3.83	2	64	76	68	4.21	.38
19	2	80	72	84	3.79	2	70	79	72	4.16	.37
20	2	85	75	90	3.75	2	76	82	76	4.1	.35
21	2	90	78	96	3.71	2	82	85	80	4.05	.34
22	2	95	81	102	3.68	2	88	88	84	4.	.32
23	1	97	87	102	3.78	2	93	91	90	4.03	.25
24	1	99	93	102	3.87	2	99	94	94	4.12	.25
25	1	101	99	102	3.96	2	105	97	98	4.2	.24
26	1	103	105	102	3.92	3	105	103	101	4.03	.11
27	1	105	111	102	3.78	3	105	109	104	4.03	.25
28	2	110	114	108	3.86	3	105	115	107	4.11	.25
29	3	115	118	111	3.83	3	105	121	110	4.17	.34
30	2	120	121	117	3.9	3	105	127	113	4.24	.34

APPENDIX B

THE IMPROVED BROWN METHOD PROGRAM

Add.	Instruction or Data				Comment
0400	35	0000	0100	0100	Store selected row /00X0-00X7/
0401	35	0400	0535	0400	in row sum /0100-0107/,
0402	35	0536	0531	0531	(0536) = 1 (0330) = No. of Cols.
0403	34	0330	0531	0400	
0404	34	0100	0101	0511	Select min value in row sum
0405	35	0404	0537	0404	
0406	35	0536	0532	0532	
0407	34	0333	0532	0404	
0410	32	0404	0540	2106	Prepare to tally in selected min column
0411	30	2006	0541	2005	
0412	30	2005	0541	2007	
0413	35	2005	2007	2007	
0414	35	0507	2007	0423	
0415	32	0404	0542	2103	Prepare to add selected column to column sum
0416	35	2006	2007	2007	
0417	35	0510	2006	0424	
0420	34	3000	2100	0421	
0421	35	0543	2100	0430	
0422	35	2003	2100	0327	Save selected column
0423		0			Tally in selected column
0424		0			
0425	35	0424	0544	0424	Add selected column to column sum
0426	35	0536	0533	0533	
0427	34	0331	0533	0424	
0430		0			Select max of column sum
0431	35	0430	0545	0430	
0432	35	0536	0534	0534	
0433	34	0334	0534	0430	
0434	32	0430	0546	2106	Prepare to tally in selected row max
0435	30	2006	0547	2005	
0436	30	2006	0541	2007	
0437	35	2006	2005	2003	
0440	35	0550	2100	0404	Reload min selector
0441	30	2005	0556	2003	
0442	35	0551	2003	0400	Reload selected row
0443	35	2006	2007	2004	
0444	35	2100	2100	2003	
0445	35	2003	2004	2004	
0446	35	0553	2004	0164	Save row tally
0447	32	0430	0554	2105	

0450	35	2005	2100	0337	
0451	17	2100	2100	0560	TS to print $\bar{v}-\underline{v}$ in improved method
0452	26	2100	2100	0531	
0453	26	2100	2100	0533	Reset
0454	35	0536	0324	0324	Tally iteration tally
0455	35	0324	2100	2007	
0456	34	3000	2100	0457	
0457	34	3000	2100	1700	Convert iteration tally to floating point
0460	31	2000	2001	0335	
0461	35	0555	0327	0462	Convert min row sum to floating point
0462		0			
0463	34	3000	2100	1700	
0464	31	0335	0336	2002	Compute $\underline{v}$ by floating point
0465	35	0556	2100	2004	
0466	34	3000	2100	1000	
0467	31	2000	2001	0320	Store $\underline{v}$ in 0320, 0321
0470	35	0557	0337	0471	Convert max column sum to floating point
0471		0			
0472	34	3000	2100	1700	
0473	31	0335	0336	2002	Compute $\bar{v}$ by floating point
0474	35	2100	0556	2004	
0475	34	3000	2100	1000	
0476	31	2000	2001	0322	Store $\bar{v}$ in 0320, 0321
0477	31	0320	0321	2002	
0500	35	2100	2100	2004	Compute $\bar{v}-\underline{v}$
0501	34	3000	2100	1000	
0502	31	2000	2001	0325	Store $\bar{v}-\underline{v}$ in 0325, 0326
0503	17	2040	2100	0340	TS to print $\underline{v}$, $\bar{v}$, iteration $\bar{v}-\underline{v}$
0504	34	3000	2100	0110	Jump to modified routine
0505	34	3000	2100	0136	" " " "
0506	34	3000	2100	0164	" " " "
0507	35	0536	0310	0310	Reload for column tally
0510	35	0000	0200	0200	
0511	32	0404	0554	2106	Auxiliary for finding min row sum
0512	30	2006	0547	2007	
0513	35	2006	2007	2007	
0514	35	0530	2007	0404	
0515	35	0536	0532	0532	
0516	34	0333	0532	0404	
0517	34	3000	2100	0410	
0520	32	0430	0542	2106	Auxiliary for finding max col sum
0521	30	2006	0541	2007	
0522	35	2006	2007	2007	
0523	35	0527	2007	0430	
0524	35	0536	0534	0534	
0525	34	0334	0534	0430	
0526	34	3000	2100	0434	
0527	34	0001	0000	0520	Data and reloads

0530	34	0000	0001	0511	Data and Reloads (cont'd)
0531		0			
0532		0			
0533		0			
0534		0			
0535	00	0001	0001	0001	
0536				1	
0537	00	0000	0001	0000	
0540	00	0007	0000	0000	
0541	02	0000	0000	0014	
0542	00	7777	0000	0000	
0543	34	0201	0200	0520	
0544	00	0010	0001	0001	
0545	00	0001	0000	0000	
0546	00	0000	0007	0000	
0547				14	
0550	34	0100	0101	0511	
0551	35	0000	0100	0100	
0552	02	0000	0000	0003	
0553	35	0536	0300	0300	
0554	00	0000	7777	0000	
0555	35	0000	2100	2007	
0556				3	
0557	35	2100	0000	2007	
0560	21	0256	0344	0002	Print $\bar{v}$(hold) - $\underline{v}$(hold)
0561	34	3000	2100	0452	Jump back to main routine

(Modification to Brown method starts here.)

0110	33	0250	0320	0505	Is new $\underline{v}$ greater than old $\underline{v}$?
0111	33	0320	0250	0113	If not, jump back to main routine
0112	33	0251	0321	0505	
0113	35	0300	2100	0230	
0114	35	0113	0172	0113	Enter new row tally in 0230-0237
0115	35	0536	0167	0167	
0116	34	0331	0167	0113	
0117	35	2100	2100	0167	Reload 0167
0120	35	0170	2100	0113	Reload 0113
0121	35	0324	2100	0252	Put iteration tally in 0252 $\underline{N}$
0122	31	0320	0321	0250	Put new $\underline{v}$ in 0250, 0251
0123	31	0320	0321	2002	
0124	31	0253	0254	2000	Compute $\bar{v}$-$\underline{v}$ improved method
0125	35	2100	2100	2004	
0126	34	3000	2100	1000	
0127	31	2000	2001	0256	Store $\bar{v}$-$\underline{v}$ improved method in 0256, 0257

0130	17	2020	2100	0133	If 2020 on, don't print strategy max $\underline{v}$, max $\underline{N}$
0131	21	0230	0344	0010	Print Player I strategy $\underline{v}$, $\underline{N}$
0132	21	0250	0344	0003	
0133	33	0256	0332	0505	$\bar{v}-\underline{v}<\mathcal{E}$ No jump back to main routine
0134	21	0256	0344	0002	Print $\bar{v}-\underline{v}$
0135	21	0230	0344	0030	Print Player I strategy, Player II strategy $\bar{v}$, $\underline{v}$ $\underline{N}$ N
0136	33	0322	0253	0506	Is new $\bar{v}$ less than old $\bar{v}$?
0137	33	0253	0322	0141	If not, jump back to main routine
0140	33	0323	0254	0506	
0141	35	0310	2100	0240	Enter new column tally in 0240-0247
0142	35	0141	0172	0141	
0143	35	0536	0167	0167	
0144	34	0330	0167	0141	
0145	35	2100	2100	0167	Reload 0167
0146	35	0171	2100	0141	Reload 0141
0147	31	0322	0323	0253	Put $\bar{v}$ in 0253, 0254
0150	35	0324	2100	0255	Put $\bar{N}$ in 0255
0151	31	0322	0323	2000	Compute
0152	31	0250	0251	2002	$\bar{v}-\underline{v}$ improved method
0153	35	2100	2100	2004	
0154	34	3000	2100	1000	
0155	31	2000	2001	0256	Store new $\bar{v}-\underline{v}$ in 0256, 0257
0156	17	2010	2100	0161	If 2010 on, don't print
0157	21	0240	0344	0010	Print min player strategy, $\bar{v}$, $\bar{N}$
0160	21	0253	0344	0003	
0161	33	0256	0332	0506	$\bar{v}-\underline{v}<\mathcal{E}$ No jump back to main program
0162	21	0256	0344	0002	Print $\bar{v}-\underline{v}$
0163	21	0230	0344	0030	Print Player I strategy, Player II strategy $\underline{v}$ $\underline{N}$ $\bar{v}$ $\bar{N}$
0164		0			Tally max row
0165	33	0325	0332	0400	Is $\bar{v}-\underline{v}$ Brown method $<\mathcal{E}$? No back to next iteration
0166	34	3000	2100	0342	Print out Brown strategy
0167		0			$\bar{v}$, $\underline{v}$ N $\bar{v}-\underline{v}$
0170	35	0300	2100	0230	
0171	35	0310	2100	0240	
0172		0001	0000	0001	
0340	21	0320	0344	0007	
0341	34	3000	2100	0504	
0342	21	0300	0344	0030	
0343	22	0000	0000	0000	
0344	02	0000	0000	0000	

1700	27	3000	1721	2006
1701	35	1711	2006	1710
1702	35	2007	2100	2000
1703	34	2000	2100	1712
1704	36	1720	1715	2003
1705	30	2007	2003	2001
1706	35	1715	2100	2000
1707	35	2100	2100	1715
1710		0		
1711	34	3000	2100	0000
1712	30	2000	1717	2000
1713	35	1716	1715	1715
1714	34	3000	2100	1703
1715		0		
1716		1		
1717	02	0000	0000	1
1720				44
1721	02	0000	0000	0000

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